Market Dynamics and Tail Loss Optimization in the Presence of Top-of-the-Book Imbalance: A CME ES Futures Case Study.

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Abstract

Effective market-making requires real-time management of tail risk arising from adverse price movements. The study analyses the short-term price dynamics of the CME ES futures at the moments of observed top-of-the-book (TOB) quote imbalance. Our findings confirm that TOB imbalance could serve as an effective predictor of short-term market movement direction, with statistical significance validated over a 5-second post-event horizon.

To capture market state at imbalance times we introduce and examine additional factors (separately and in conjunction with imbalance) as directional predictors. We also explore a simple driftless random walk model for weighted mid-price process to estimate probability of TOB price change in the direction implicated by imbalance and observe a good match of model forecast with empirical estimates.

We formulate the market-making quote cancellation problem as a direct optimization task to minimize expected trading loss. We introduce a *percentile optimization framework* to train feed-forward neural network. The neural network model learns a multi-factor scoring function that is not only predictive but is explicitly optimized for the outcome of the cancellation policy, minimizing Conditional Value-at-Risk (CVaR) across multiple operational thresholds.

When trained on years 2018-2020 and evaluated on out-of-sample years 2021-2024, the neural-network based algorithm consistently outperformed all single-factor benchmark algorithms, proving its superiority in managing tail risk and improving P&L.

The study covers seven years of high-frequency CME ES contract data (2018–2024).

Acknowledgement

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Introduction

The application of machine learning to financial risk management has evolved from probabilistic forecasting to direct decision optimization. Pioneering work focused on quantile regression [1, 2] to model the conditional distribution of returns/losses, providing measures like Value-at-Risk (VaR). This was extended by research on Expected Shortfall (ES) / Conditional Value-at-Risk (CVaR) optimization, which directly addresses tail risk minimization [3].

Concurrently, the field of learning-to-rank [4] demonstrated the power of training models to order data based on a utility signal, rather than just predict values. In more recent publications, scientists have explored neural networks to estimate financial extremes, often by tailoring the loss function to prioritize tail accuracy.

However, existing methods mostly separate forecasting task and decision-making. A model first predicts a distribution (e.g., of loss), then a separate policy is applied to forecast. To bridge the gap, we propose end-to-end framework that directly learns a scoring function optimized for a specific operational policy. By dynamically learning thresholds and leveraging a differentiable, tail-focused loss, our method does not only predict risk - it learns to minimize it within a decision policy, unifying forecast and action into a single optimization objective.

Market data

This study utilizes seven years (2018–2024) of discretely sampled best bid/offer (BBO) market data for the CME ES futures continuous contract, with a sampling frequency of one second.

Derived time-series indicators

Using the discretely resampled one-second TOB snapshots, we compute the following auxiliary indicators to capture short-term price dynamics of the market:

Table 1. indicators to capture short term market dynamics

Indicator name	Description
Liq—TOB side	The less liquid TOB side (BID or ASK): illiquid side
Liq++ TOB side	The more liquid TOB side (BID or ASK): liquid side
Imbalance	TOB size imbalance calculated as: (bid size - ask size)/(bid size + ask size)
Average bid/ask size	Exponentially weighted moving average of bid and ask sizes with a decay period N = 120.
Norm Liq-- size	Ratio of the current illiquid TOB size to its EMA value
Norm Liq++ size	Ratio of the current liquid TOB size to its EMA value
Weighted mid-price	Size-weighted mid-price: w = bid size / (bid size + ask size) mid-price = w * ask price + (1-w) * bid price
Historical volatility *v5s1* *v60s1*	EMA-based volatility calculated from 1-second log returns of weighted mid-price (in basis points): v5s1 with decay period N = 5 v60s1 with decay period N = 60
Z-score	EMA based version of z-score indicator with period N = 3600

Momentum	Momentum (over past N seconds) adjusted by volatility: N = 1,2,5,15,30

Construction of imbalance sample dataset

Even one-second TOB snapshot dataset is still very large, comprising approximately 20 million observations per year (250 trading days × 23 hours × 3,600 seconds). To reduce this to a more manageable dataset size while preserving statistical significance, we apply the following procedures:

- **Imbalance threshold filtering:** Exclude cases where the absolute imbalance is below 0.5. This threshold corresponds to a moderate liquidity imbalance (liquid-to-illiquid size ratio of 3:1).
- **Trading hours filtering:** Retain data only from 6:30 PM to 4:05 PM NY time (the next day) to exclude non-liquid session start/end time.
- **Random sampling with stratification:** Due to the non-uniform density of imbalance, we employ a floating acceptance threshold to ensure sufficient representation across all imbalance bins and prevent under-sampling of some bins.

Capturing post event price dynamics

For each imbalance event selected by our filtering procedure, we collect several output variables to quantify the impact of the imbalance and/or other factors on price dynamics over a 5-second post-event horizon.

We differentiate between the liquid (Liq++) and illiquid (Liq--) sides of the TOB. Our analysis primarily focuses on the illiquid side, as it presents a decision point for trading algorithms:

- **Market Maker**: Whether to maintain or cancel quotes on the illiquid side, given the heightened risk of unrealized losses within seconds.
- **Arbitrary Execution Algorithm**: An opportunity to switch from pegging on passive side to execution by market if a favourable price movement is anticipated.

Output Variables

For each imbalance event, we compute the post-event metrics listed in Table 2. The interpretation of these metrics may depend upon the role of market participants, e.g.:

- **Market Maker:** When keeping a quote on the illiquid best side (Liq—), positive post-event forward P&L, as we calculate it, means an unrealized loss for market maker, negative P&L means gaining an unrealized profit.

Table 2. Performance metrics to capture post-event price dynamics.

Name	Description
Forward P&L Liq—(bps)	Profit/loss (in basis points) for the Liq—TOB side at the end of post-event time interval, e.g., 5 seconds. Positive, if the side has been moved in the direction implied by imbalance.
Forward P&L Liq++ (bps)	Same as above, but for the liquid side.
Forward price change direction Liq--	Captures price change direction of the illiquid side at the end of post-event time interval: +1 if the price moved in the direction implied by the imbalance. -1 if adverse movement. 0 if unchanged.
Forward price change direction Liq++	Same as above, but for the liquid side.
First price change direction Liq--	Captures the *initial* price movement within the 5-second post-event time window: +1 if the illiquid-side price moves first in the imbalance-implied direction. -1 if adverse. 0 if no change.
First price change direction Liq++	Same as above but for the liquid TOB side.

CME ES TOB liquidity and spread trading profitability

The CME ES futures contract is a highly liquid instrument, typically trading at a one-tick spread between the best bid and ask prices. However, two trends have emerged over the study period (2018–2024):

- **Declining profitability of spread trading (in basis points)**: The tick size remains fixed at $0.25, while the S&P 500 index has grown consistently (except of year 2022) causing decrease of the profitability of spread trading (measured in basis points).
- **Reducing TOB Liquidity (in a number of contracts)**: The average number of contracts available at the TOB has exhibited a year-over-year decline over the study period (2018-2024).

In Table 3 we provide statistics of distribution of the spreads (in ticks) across our sample dataset (which record market state at imbalance time) .

Table 3. Per annum TOB spread distribution (in ticks) in the sample dataset.

Spread (ticks)	2018	2019	2020	2021	2022	2023	2024
1	486,051	435,688	600,610	570,049	573,748	520,694	463,240
2	2,135	176	15,359	518	4,502	368	1,499
3	86	2	1,972	11	64	27	46
4	17	2	604	4	18	10	6
5	4		178	1	5	5	3
6	1		74			1	2
7	1		17				
TOTAL	**488,295**	**435,868**	**618,814**	**570,583**	**578,337**	**521,105**	**464,796**

Other liquidity metrics and their year-of-year dynamics are provided in Table 4.

Table 4. Year-of-Year TOB liquidity metrics (2018-2024) at imbalance times

Metrics	2018	2019	2020	2021	2022	2023	2024
Avg Spread (bps)	0.92	0.87	0.82	0.53	0.62	0.59	0.46
Avg Contract Price	2,727.96	2,894.07	3,202.77	4,257.29	4,100.23	4,278.86	5,461.97
Avg Size Liq-- Side	16.58	18.20	9.01	14.38	4.95	9.25	8.29
Avg Size Liq++ Side	106.90	108.04	59.22	147.00	31.66	56.60	48.60

Observed facts have direct implications for CME ES market participants as narrowing spreads and declining contract sizes pressure spread trading profitability.

Estimate of empirical probability of price change

For each observed imbalance event, we classify the 5-second post event time price change as:

Table 5. Forward price movement direction

Direction (D)	Condition
D = +1	Price of TOB side (bid or ask) changes in the direction predicted by imbalance (forecast match)
D = -1	Price of TOB side (bid or ask) changes in adverse direction to predicted by imbalance
D = 0	Price remains unchanged

We separate calculation of price change direction for illiquid and liquid sides of the TOB. We will focus mostly on the dynamics of the illiquid side of the TOB as this is a decision point for:

- **Market maker:** either to keep quote or cancel it.
- **Execution algorithm:** potentially to turn into aggressive execution by market.

Probability Estimate

We will aggregate our imbalance cases into the buckets (single or two dimensional) and estimate probability of forecast match or adverse move using indicator variables calculated for all samples in the bucket.

Modelling weighted mid-price process by random walk

Let's consider discretionary sampled (Δt = 1 sec) weighted mid-price process:

$$P(t) = \frac{Price_{bid}(t) \cdot Size_{ask}(t) + Price_{ask}(t) \cdot Size_{bid}(t)}{Size_{ask}(t) + Size_{bid}(t)} \qquad (1)$$

Benefits of choosing the weighted mid-price for liquid contract (with a lot of order book updates during a single second):

- **Smoother price path:** It generates a much smoother price path comparing to traditional mid-price. For example, it will not "jump" as much as classic mid-price when an aggressive order is erasing the illiquid side of TOB. The smoothness means increments of the weighted mid-price are much smaller and much more frequent than of traditional mid-price.
- **Random Walk assumption:** Because the individual increments are smaller and frequent (>> 1 per second), the process of aggregating these small changes over a fixed size window results to a large number of small independent steps. Then due to Central Limit Theorem returns of discretionary sampled pricing process converge to normal distribution and the process itself could be modelled by random walk (RW).

By estimating parameters of random walk and calculating minimal distance the weighted mid-price shall move to ensure TOB price change in the direction implied by imbalance, we will be able to estimate model probability of TOB price change and compare it against probability estimated from the market data.

Parameters of random walk

Drift (μ)

We assume the drift μ is negligible on the 5 second time horizon.

Volatility (σ)

We estimate exponentially averaged historical volatility $\sigma(t)$ of 1 second log returns of the weighted mid-price process P(t). We assume that volatility estimate is constant during post event time horizon.

Probability of random walk to terminate above the barrier α

For the driftless random walk process *P(t)* with 1 second volatility $\sigma(t)$ and $P = 0$ at the imbalance time $t = 0$, a probability of the process to end above the threshold α at time t is given by:

$$\text{Prob}(P(t) \geq \alpha) = 1 - \phi\left(\frac{\alpha}{\sigma\sqrt{t}}\right) \quad (2)$$

Where Φ is a standard normal CDF, σ is 1 second volatility estimate, t = 5 is future time horizon from the event time.

Barrier estimate

This is trickier parameter to estimate on discretionary sampled data. Ideally, we shall do it using raw top-of-the-book price data.

Let's first note that we are looking for such nearest future weighted mid-price change which corresponds to the TOB price change in the direction of imbalance. We also know (Table 3) that most likely TOB price spread after the change will be 1 tick. Then our distance will be a distance to the illiquid (**Liq--**) price level which is rounded to tick plus some *eps* which is defined by extreme TOB size configuration after the change (Figure 1).

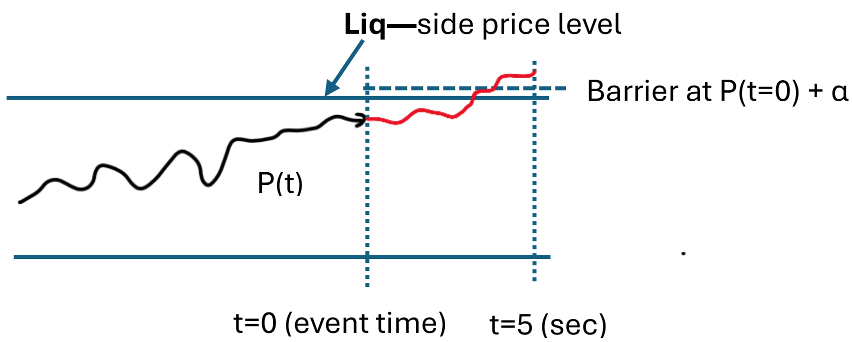


Figure 1. Hitting barrier by RW process

Let's estimate *eps* assuming extreme imbalance configuration after TOB price change.

For imbalance 0.95 which correspond to liquid/illiquid size ratio 39:1, the *eps* could be directly calculated from formula (1) and results to 1/40 of tick.

For the average number of contracts on liquid side for year 2023 (Table 4) – 56.6, the number of contracts on illiquid side, corresponding to imbalance 0.95 right after TOB price change, will be slightly less than 1.5 contracts.

Exploring relationships between input variables and performance metrics

To explore relationships between our indicators selected to describe a market state at imbalance point (Table 1) and performance metrics (Table 2), we will apply simple but quite efficient method called bucketing analysis. Step-by-step procedure includes:

(1) Choose an input variable to explore, e.g., *imbalance*, *norm Liq-- size*, *volatility*, or combination of variables and performance metric.
(2) Split the min-max range of the input variable into intervals (buckets). We will use these two methods:
   - *Equal width buckets*: Divides the range into equal size buckets. For example, positive values of the imbalance could split into 0.5-0.6, 0.6-0.7, ...
   - *Equal frequency buckets:* Divides the data into N buckets where each bucket has approximately the same number of samples.
(3) Aggregate cases into the buckets and calculate the average value of the selected metric(s).
(4) Visualize and analyse results.

Bucketing Analysis - Imbalance

- **Input**: Imbalance indicator from the Table 1.
- **Buckets**: Our data set includes cases with | imbalance | > 0.5. Imbalance only has upper limit 1.0. The step we use for aggregating our cases is 0.1
- **Performance metrics**: For each bucket we calculate these metrics:
   - *Case count:* Number of cases in the bucket.
   - *Average value of the forward P&L Liq— side:* This is average value of the *forward P&L Liq—* side available for each imbalance case (see Table 2).
   - *Average value of the forward P&L Liq++ side:* This is average value of the *forward P&L Liq++* side available for each imbalance case (see Table 2).
   - *Liq—side First Price Change*: We estimate two probabilities from the data using *first price change direction Liq—*variable (see Table 2).
      - *Match forecast probability*: This is estimated by dividing a number of cases in the bucket for which the given variable takes a value 1 to the total number of cases in the bucket.
      - *Adverse move probability*: This is estimated by dividing a number of cases in the bucket for which the given variable takes a value -1 to the total number of cases in the bucket.
   - *Liq—side End of Interval Price Change*: We estimate two probabilities from the data using *forward price change direction Liq-- variable* (see Table 2).
      - *Match forecast probability*: This is estimated by dividing total number of bucket cases for which this variable takes a value 1 to the total number of cases in the bucket.

- *Adverse move probability*: This is estimated by dividing total number of cases in the bucket for which the given variable takes a value -1 to the total number of cases in the bucket.

- *Random walk probability of matching forecast:* To calculate this probability for each case in the bucket (for illiquid side of the TOB Liq--), we use historical volatility estimate on 1 second periodic log-returns of weighted mid-price (Table 1). The barrier is estimated by procedure described above. The probability is calculated by formula (2).

Analysis results

The results for year 2023 are provided in the table below. The results for 7 years 2018-2024 are available in **Appendix 1**.

Imbalance buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: First price change direction within 0<=t<=5 sec from event time				Liq-- side: Forward price change direction at t = 5 sec from event time				Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec)
0.9	1	0.95	35,023	0.255	0.224	22,718	3,715	0.65	0.106	16,331	4,154	0.466	0.119	0.464
0.8	0.9	0.85	54,288	0.237	0.207	32,440	6,229	0.60	0.115	23,984	6,864	0.442	0.126	0.439
0.7	0.8	0.75	58,004	0.208	0.183	31,175	7,409	0.54	0.128	23,926	7,863	0.412	0.136	0.410
0.6	0.7	0.65	58,186	0.186	0.165	28,704	8,043	0.49	0.138	22,438	8,432	0.386	0.145	0.384
0.5	0.6	0.55	55,688	0.167	0.149	25,323	8,612	0.45	0.155	20,530	8,541	0.369	0.153	0.360
-0.5	-0.6	-0.55	55,389	0.158	0.139	24,954	8,563	0.45	0.155	19,806	8,717	0.358	0.157	0.362
-0.6	-0.7	-0.65	58,604	0.181	0.160	28,588	8,034	0.49	0.137	22,299	8,505	0.381	0.145	0.386
-0.7	-0.8	-0.75	57,415	0.211	0.185	31,193	7,079	0.54	0.123	23,492	7,660	0.409	0.133	0.411
-0.8	-0.9	-0.85	53,743	0.234	0.204	31,724	6,325	0.59	0.118	23,462	6,942	0.437	0.129	0.440
-0.9	-1	-0.95	34,765	0.259	0.227	22,159	3,680	0.64	0.106	15,975	4,118	0.460	0.118	0.465
			521,105	0.206	0.181	278,978	67,689	0.54	0.130	212,243	71,796	0.407	0.138	**0.407**

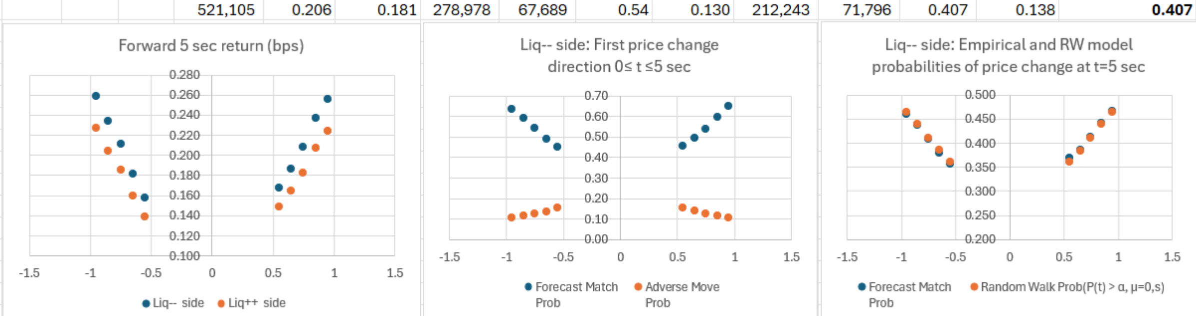


Figure 2. Dependency of performance metrics from the TOB size imbalance – Bucketing analysis results for 2023

Forward 5 sec unrealized P&L (bps) plot

On this plot we observe close to linear dependency of forward unrealized P&L from the imbalance. The more extreme is TOB side imbalance the larger expected forward P&L.

This is true for both illiquid and liquid sides of the TOB.

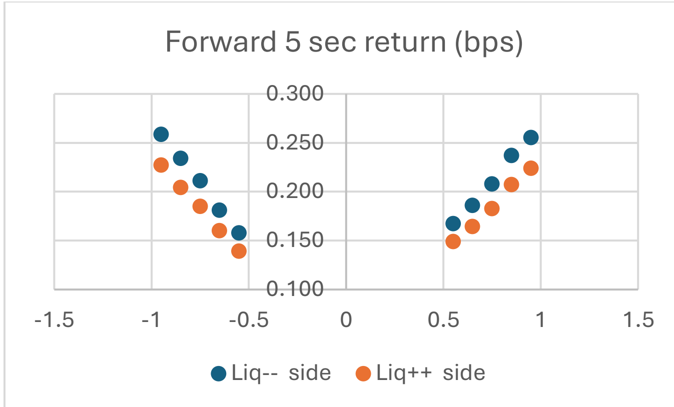


Figure 3. Dependency plot of forward return (P&L) from the imbalance - Year 2023

Let's note that for extreme buckets [0.9 to 1.0] and [-0.9 to -1.0] the forward P&L expected value in 2023 is roughly 0.26 bps, looking at average tick spread value 0.59 bps for 2023 (see Table 4), we can estimate expected loss of the market maker who keeps its quote/order on the illiquid side. The expected loss in presence of such extreme imbalance will be approximately 40% of the spread.

Empirical vs RW model probability plot

The plot shows dependency of both: (i) empirical probability (matching imbalance forecast) for illiquid side of the TOB; and (ii) RW model probability upon TOB imbalance value.

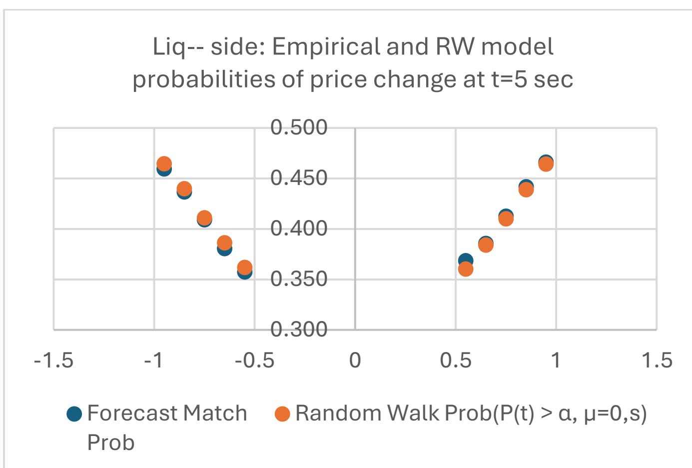


Figure 4. Dependency of empirical forecast match probability of the illiquid side of TOB and RW model probability from imbalance. Year 2023

The fact that a simple model based on a pure theoretical construct closely matches empirical probabilities highly likely indicates that the core driver of *directional changes* at the top of the book in presence of the imbalance is the random arrival/cancellation of the quotes/orders from market participants.

RMSE of residuals of two probabilities for 2023 RMSE is 0.006, this is good match and provides strong empirical support for the Random Walk model applicability at the microstructure level. The results for 7 years 2018-2024 are available in **Appendix 1**.

Bucketing analysis - Volatility

Let's estimate impact of volatility on unrealized P&L and probability of illiquid side price to change in the direction indicated by imbalance.

Let's group all imbalance cases in our dataset into 10% deciles by volatility and estimate these two metrics for each bucket.

Note: As Volatility estimator we will use *v60s1* which is EMA based volatility estimator on one second log returns of the weighted mid-prices with EMA averaging period = 60.

Bucket analysis of volatility impact. Results for year 2023

The results for year 2023 are provided in the table below. The results for all 6 years 2018-2024 are provided in **Appendix 2**.

Forward 5 sec unrealized P&L and probability of TOB price change in the direction forecasted by imbalance are low for law volatility and increased for higher volatility.

Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.48	8.15	0.90	52,111	0.222	0.196	24,602	17,505	0.472	0.336
0.35	0.48	0.80	52,110	0.214	0.195	23,712	13,458	0.455	0.258
0.28	0.35	0.70	52,111	0.208	0.187	22,774	10,749	0.437	0.206
0.23	0.28	0.60	52,110	0.204	0.179	22,089	8,577	0.424	0.165
0.20	0.23	0.50	52,111	0.208	0.181	21,496	6,707	0.413	0.129
0.17	0.20	0.40	52,110	0.204	0.176	20,564	5,173	0.395	0.099
0.14	0.17	0.30	52,110	0.207	0.180	20,288	3,742	0.389	0.072
0.12	0.14	0.20	52,111	0.201	0.174	19,649	2,855	0.377	0.055
0.10	0.12	0.10	52,110	0.199	0.172	19,146	2,002	0.367	0.038
0.02	0.10	0.00	52,111	0.188	0.166	17,923	1,028	0.344	0.020
			521,105	0.206	0.181	212,243	71,796	0.407	0.138

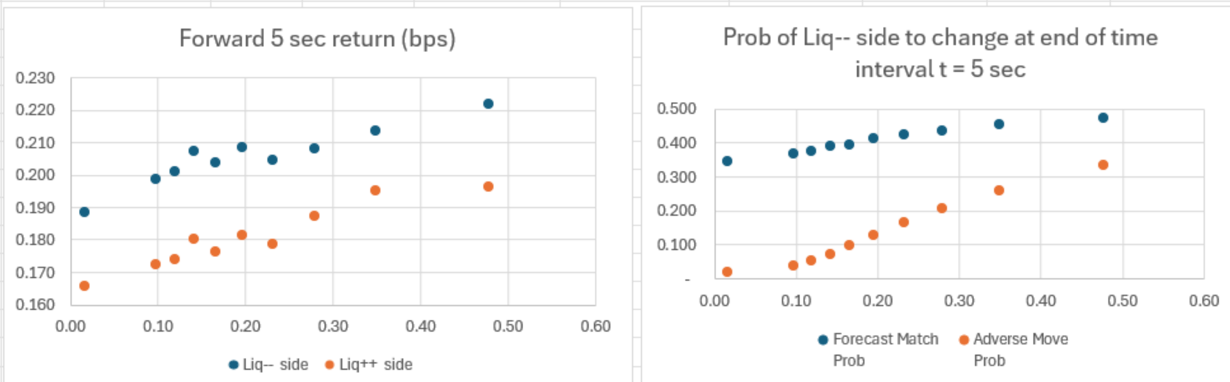


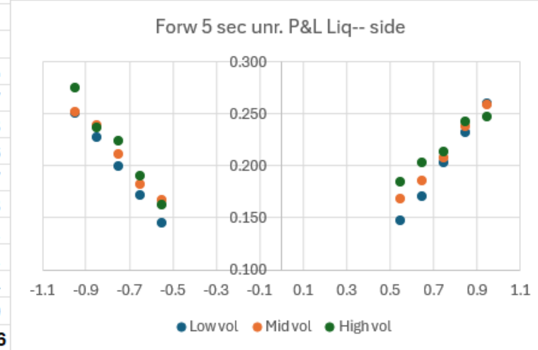
Figure 6. Dependency charts of forward return and probability of illiquid side of the TOB to erase from market volatility

Two-dimensional bucket analysis by Imbalance and Volatility factors

The results for year 2023 are provided below. The results for all 6 years 2018-2024 are provided in **Appendix 3**.

For Volatility we used these buckets: 0 to 0.333, 0.333 to 0.667, 0.667 to 1.0

Forw 5 sec unr. P&L Liq-- side						
		Volatility	0.149	0.262	8.151	
Imbalance Buckets		Buckets	0.017	0.149	0.262	
0.9	1	0.95	0.260	0.259	0.247	0.255
0.8	0.9	0.85	0.232	0.238	0.242	0.237
0.7	0.8	0.75	0.203	0.208	0.213	0.208
0.6	0.7	0.65	0.170	0.185	0.203	0.186
0.5	0.6	0.55	0.147	0.169	0.184	0.167
-0.5	-0.6	-0.55	0.145	0.167	0.162	0.158
-0.6	-0.7	-0.65	0.171	0.182	0.190	0.181
-0.7	-0.8	-0.75	0.200	0.211	0.223	0.211
-0.8	-0.9	-0.85	0.227	0.239	0.236	0.234
-0.9	-1	-0.95	0.251	0.251	0.275	0.259
			0.197	0.207	0.213	**0.206**



Prob of Liq-- price change in direction matching imbalance						
		Volatility	0.149	0.262	8.151	
Imbalance Buckets		buckets	0.017	0.149	0.262	
0.9	1	0.95	0.457	0.463	0.480	0.466
0.8	0.9	0.85	0.417	0.438	0.471	0.442
0.7	0.8	0.75	0.375	0.411	0.453	0.412
0.6	0.7	0.65	0.327	0.384	0.445	0.386
0.5	0.6	0.55	0.292	0.369	0.438	0.369
-0.5	-0.6	-0.55	0.287	0.360	0.421	0.358
-0.6	-0.7	-0.65	0.327	0.374	0.439	0.381
-0.7	-0.8	-0.75	0.368	0.409	0.452	0.409
-0.8	-0.9	-0.85	0.410	0.434	0.467	0.437
-0.9	-1	-0.95	0.444	0.453	0.483	0.460
			0.365	0.405	0.452	**0.407**

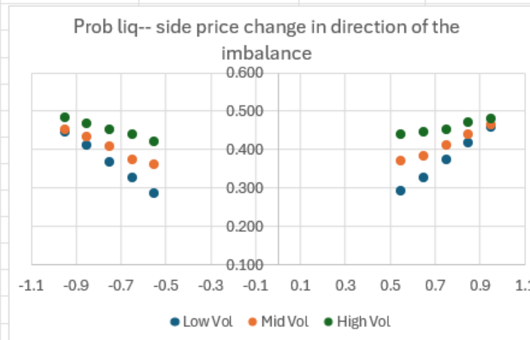


Figure 7. 2D dependency charts of forward return and probability of illiquid side of the TOB to erase from the imbalance x market volatility

Liq—normalize size factor

This is a ratio of the TOB illiquid side size at the time of the event to the average size over past two minutes of this side of the book. Intuitively, this variable should correlate with TOB imbalance

Bucket analysis by **Liq--normalized size** factor. Results for year 2023

The results for year 2023 are provided in the table below. The results for all 6 years 2018-2024 are provided in **Appendix 4**.

Both unrealized P&L and probability of illiquid TOB side price change are decreasing when **Norm Liq--** side size is increasing.

Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.44	2.95	0.90	52,111	0.135	0.121	18,311	8,955	0.351	0.172
0.37	0.44	0.80	52,110	0.165	0.146	19,178	7,822	0.368	0.150
0.32	0.37	0.70	52,111	0.185	0.165	19,761	7,412	0.379	0.142
0.28	0.32	0.60	52,110	0.190	0.169	20,096	7,185	0.386	0.138
0.24	0.28	0.50	52,111	0.200	0.176	20,706	7,124	0.397	0.137
0.19	0.24	0.40	52,110	0.213	0.187	21,525	6,871	0.413	0.132
0.15	0.19	0.30	52,110	0.225	0.197	22,070	6,854	0.424	0.132
0.11	0.15	0.20	52,111	0.231	0.200	22,673	6,818	0.435	0.131
0.07	0.11	0.10	52,110	0.254	0.220	23,700	6,559	0.455	0.126
0.01	0.07	0.00	52,111	0.258	0.226	24,224	6,196	0.465	0.119
			521,105	0.206	0.181	212,244	71,796	0.407	0.138

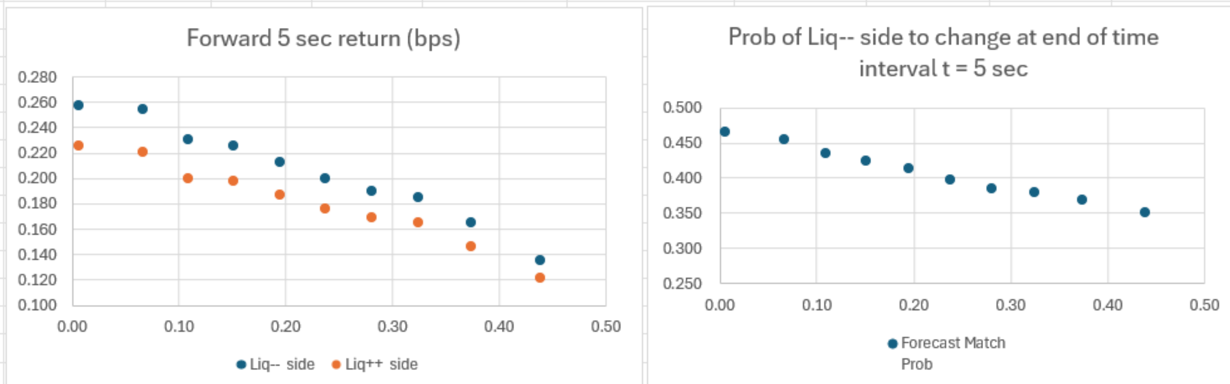


Figure 8. Charts of forward return and probability of illiquid side of the TOB to erase depending upon a market volatility

Benchmark algorithms

In this section we will compare three naïve MM algorithms that on every imbalance event taking place shall make a decision either to continue to quote the market on the illiquid side of the TOB or cancel the quote.

The "scoring" factors are:

- TOB imbalance.
- Normalized size of the illiquid side of TOB.
- RW model probability of price change of the illiquid side in the direction implied by imbalance.

Assuming more extreme losses correlate with lower values of the scoring factor, we can calculate an expected average loss which cancels quotes when scoring factor value for the sample is below some threshold:

$$\mathcal{L}(c) = \boldsymbol{E}[Y|S \geq Q_s(p)] \qquad (5)$$

where:

- c is a cancellation rate (e.g., 0.1 for 10%)
- $p = c$ is a target quantile for the scoring factor
- S is a scoring factor used to decide either to cancel the quote or keep it
- $Q_S(p)$ is the *p-th* quantile of the S distribution
- Y is P&L (loss) in bps at the end of the time interval since the imbalance event.

Calculating the average loss $\mathcal{L}(p)$ for different cancellation rates we build performance curve for the algorithm utilizing the given scoring factor.

Forward loss (Y) dependency on time interval duration

Let's estimate average forward P&L/loss (in basis points) at the end of 1, 3, and 5 second time intervals since the event time. Then for the years 2018-2024 we observe the following:

Δt(sec) Year	1	3	5
2018	0.238	0.308	0.323
2019	0.229	0.297	0.318
2020	0.248	0.275	0.277
2021	0.197	0.225	0.231
2022	0.226	0.240	0.242
2023	0.191	0.220	0.226
2024	0.145	0.164	0.169

Figure 9. YoY average forward P&L (MM loss) on 1-, 3-, and 5-sec time horizon from imbalance event

Considering loss at the end of 5 second time interval as 100% we observe that most of the loss takes place by the end of the first second with an evident loss decay after 3rd second:

Δt(sec) Year	1	3	5
2018	73.81%	95.30%	100.00%
2019	71.90%	93.41%	100.00%
2020	89.45%	99.23%	100.00%
2021	85.38%	97.58%	100.00%
2022	93.05%	98.98%	100.00%
2023	84.32%	97.04%	100.00%
2024	85.80%	97.09%	100.00%

Figure 10. YoY average loss attribution on 1-, 3-, and 5-sec time horizon from the imbalance event

Dataset for algorithm performance comparison

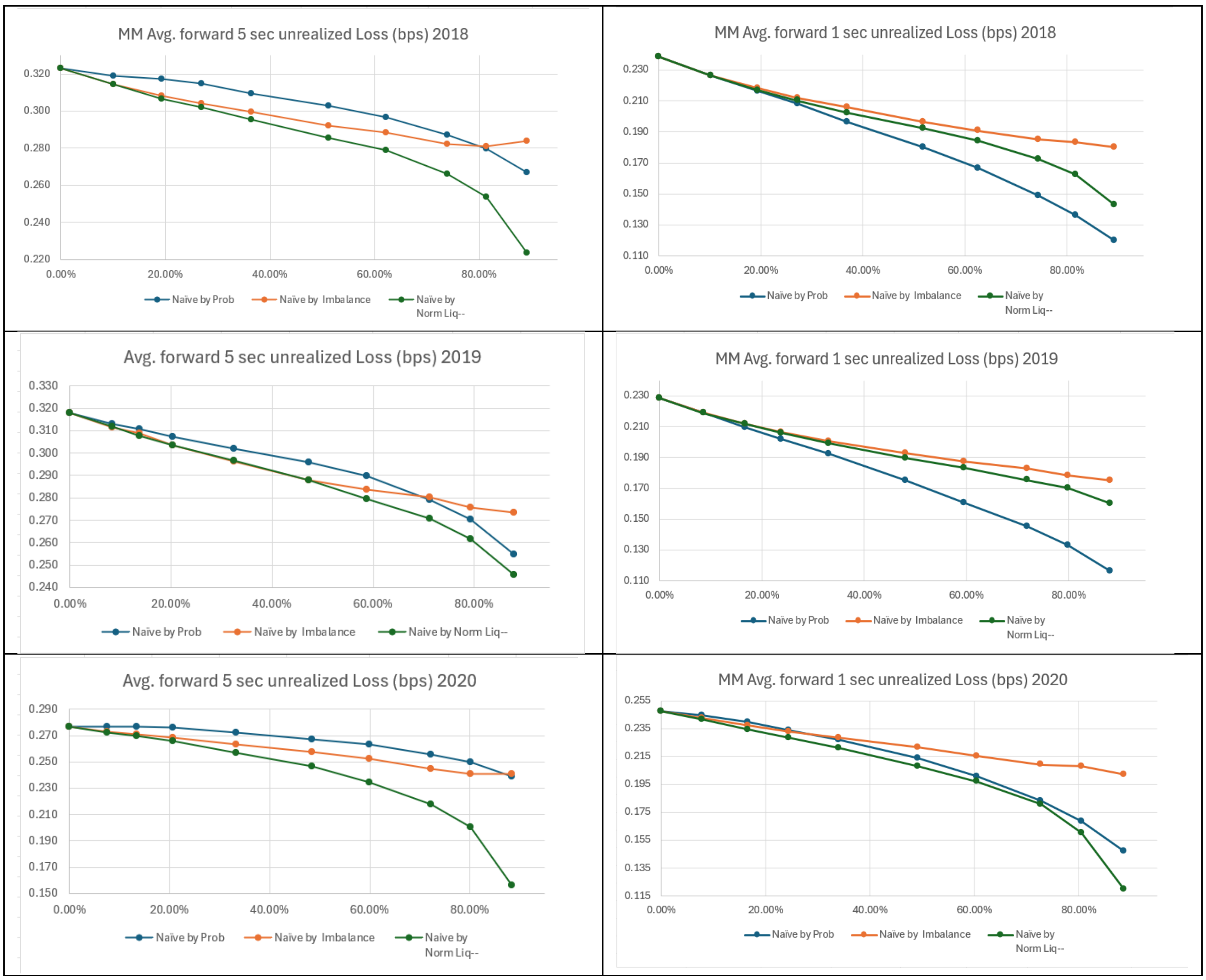
We will review/compare algorithms on years 2018-2024. Sample size for every year could vary in the range 300,000-500,000 imbalance cases selected per year.

We accept only samples with | imbalance | > 0.7 excluding from consideration cases with low or medium imbalance. Random sampling with floating acceptance threshold allows us to achieve target annual dataset size.

For each sample accepted we collect at minimum three scoring factors mentioned above and forward P&L in bps/loss estimated on 1-second and 5-second time interval since the event time.

Scoring factor performance results 2018-2024

The results for the years 2018-2024 for the algorithm utilizing the scoring factors we've chosen are provided below in the form of performance curve to demonstrate the trade-off between cancellation rate (X-axis) and average forward loss in bps (Y axis) of the algorithm.



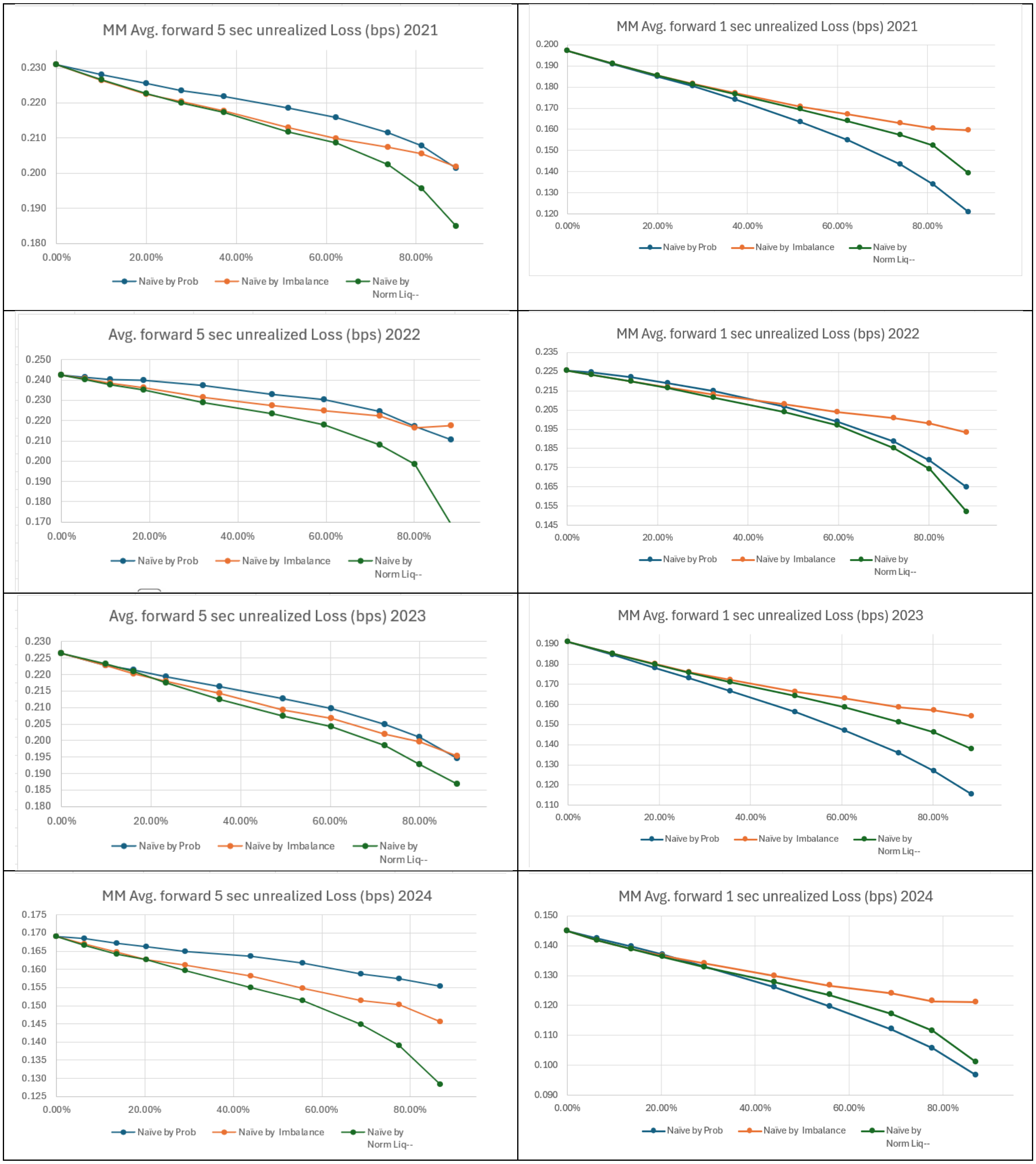


Figure 11. Forward average loss at different cancellation rates of the naïve algorithms utilizing different scoring factors. Years 2018-2024, 5-sec and 1-sec forward time intervals.

Observations:

- On 5 second time interval the best performance, i.e., a minimal loss under various cancellation conditions is demonstrated by the algorithm utilizing the **Norm Liq—** scoring factor. It consistently outperforms two others on all years 2018-2024.

- On 1 second time interval the algorithm utilizing RW model probability demonstrates best performance on all years except of 2020 and 2022 which both are characterized by higher-than-average market volatility. On these two years the algorithm using **Norm Liq—** scoring factor provides best performance.

Learning a scoring function with the neural network

Let's formalize scoring function learning task as an optimization problem:

- We would like neural network (NN) to learn scoring function $S = f(\boldsymbol{x})$, where $\boldsymbol{x}$ is a vector of input features known at the time of the imbalance event. The left tail of scoring function corresponds to more extreme state of the market and is expected to concentrate larger losses Y (forward P&Ls)
- For a chosen cancellation rate $c = p$ $(0 \leq p < 1)$, the trading policy is to cancel the quote on the illiquid side of the TOB if $S < Q_S(p)$, which is the *p-th* quantile of the distribution of scoring factor S. Otherwise to continue to keep the quote on the market.
- The **average loss** $\mathcal{L}(c)$ $of\ the\ algorithm\ implementing\ our\ trading\ policy$ is:
-

$$\mathcal{L}(c) = \boldsymbol{E}[Y|S \geq Q_s(p)]$$

- The goal is to minimize the sum of the average losses for a set of cancellation rates.

The optimization task

Find the function $f(\boldsymbol{x})$ (parameterized by the NN) that minimizes the weighted sum of average losses across a set of cancellation rates $\{c_k\}$ or equivalently, quantiles $p_k = c_k$:

$$\min_f \sum_{k=1}^{K} w_k \cdot \mathbf{E}[Y|S \geq Q_s(p_k)] \qquad (6)$$

where w_k are the weights optionally used to prioritize the losses.

NN based optimization framework

To solve the formulated optimization problem, we will explore feed forward neural networks (FFNN).

Framework Architecture

- **Input**: features $\mathbf{x} \in \mathbb{R}^n$ where **x** includes raw imbalance, normalized size of TOB illiquid side, RW probability estimate for the illiquid side to erase, volatility estimates. The **x** could optionally be extended with various non-linear transformations of raw features tailored for particular needs.
- **Network**: Standard feed-forward NN (FFNN) with a few layers and scalar output will be used to learn scoring function:

$$f_\theta : \mathbb{R}^n \to \mathbb{R}, \qquad \hat{y} = f_\theta(\boldsymbol{x})$$

- **Label variable *Y*:** this is unrealized forward P&L/loss of MM of illiquid TOB side in basis points. We use two intervals to estimate forward losses: 1and 5 second.

- **Percentile thresholds**: It is not clear how to choose thresholds for scoring function of unknown shape, so we re-calculate/adjust them dynamically after forward pass of the training cycle using NN output distribution $\hat{Y}$ (the scoring function):

$$q_k = Quantile\big(\hat{y}_j, p_k\big), p_k \in \{0.1, 0.2, \dots, 09\}$$

  To improve stability of these thresholds a smoothing procedure could be applied.

- **Loss function:** For each percentile p_k and its threshold q_k, we compute the following loss function for the sample batch of size *N*:

$$\mathcal{L}_k = \sum_{j=1}^{N} \frac{y_j}{1 - p_k} \cdot \sigma\left(\frac{\hat{y}_j - q_k}{\tau}\right)$$

  where the first term y_j is the unrealized forward P&L/loss of the sample, and the second term is a differentiable approximation of our trading policy P_{kj} with respect to sample *j* and threshold q_k:

$$P_{kj} = \sigma\left(\frac{\hat{y}_j - q_k}{\tau}\right), \quad \sigma(z) = \frac{1}{1 + e^{-z}}$$

  where $\sigma(z)$ is sigmoid function, and $\tau > 0$ is its parameter controlling the sigmoid transition sharpness from 0 to 1.

  Total loss is then computed as:

$$\mathcal{L} = \sum_{k=1}^{K} \mathcal{L}_k$$

- **Total loss gradient:** The gradient of the loss function $\mathcal{L}_k$ w.r.t scoring function output:

$$\frac{\partial \mathcal{L}_k}{\partial \hat{y}_j} = \frac{y_j}{\tau \cdot (1 - p_k)} \cdot \sigma\left(\frac{\hat{y}_j - q_k}{\tau}\right)\left(1 - \sigma\left(\frac{\hat{y}_j - q_k}{\tau}\right)\right)$$

NN training

We split available sample data set on training/validation set and true out-of-sample set.

Years used for NN model training are 2018-2020.

Years used for out-of-sample run are: 2021-2024.

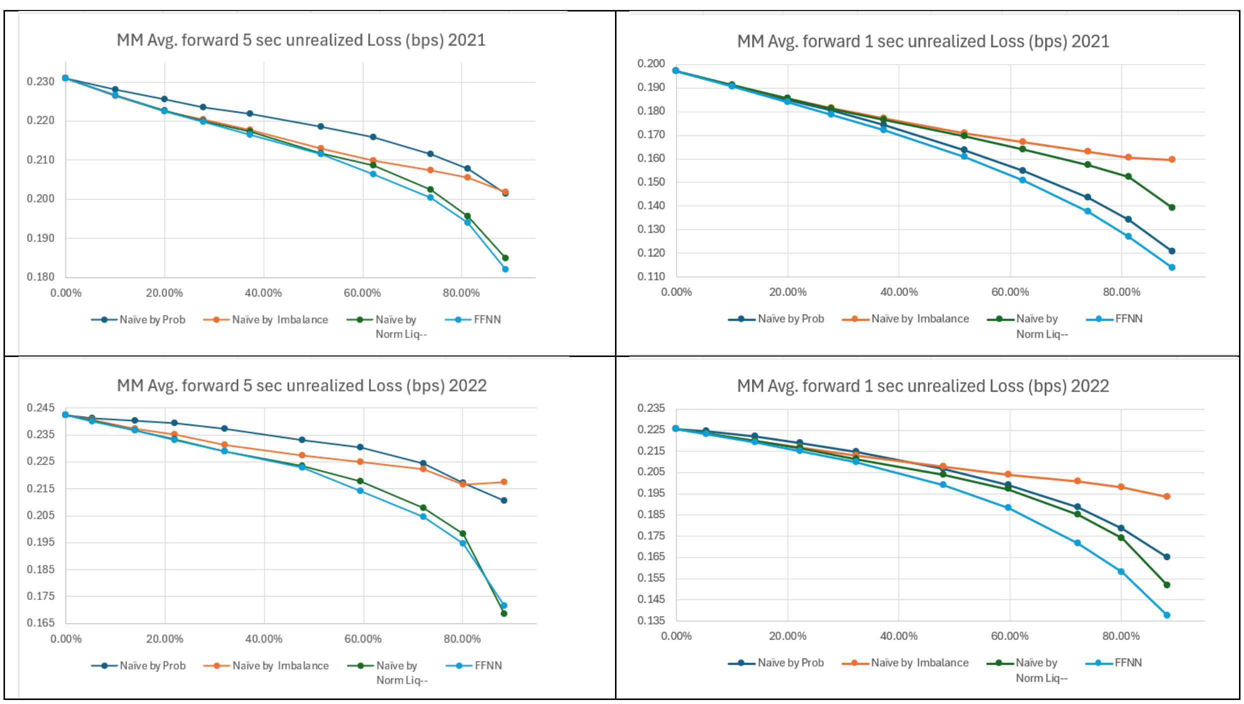
The parameters of NN found during training phase are not changed during full out-of-sample run. Model re-training will likely improve performance.

We also separately train NN on 1-second and 5-second forward returns.

Out-of-sample performance of the NN-based scoring function

Results 2021-2024

The results for the years 2018-2024 are provided in the form of performance curve to demonstrate trade-off between cancellation rate (X-axis) and average forward loss in bps (Y axis) of the algorithm. The charts provide results of three different scoring factor-based algorithm implementing our trading policy and neural network-based scoring function (**FFNN**) learned from the data 2018-2020.



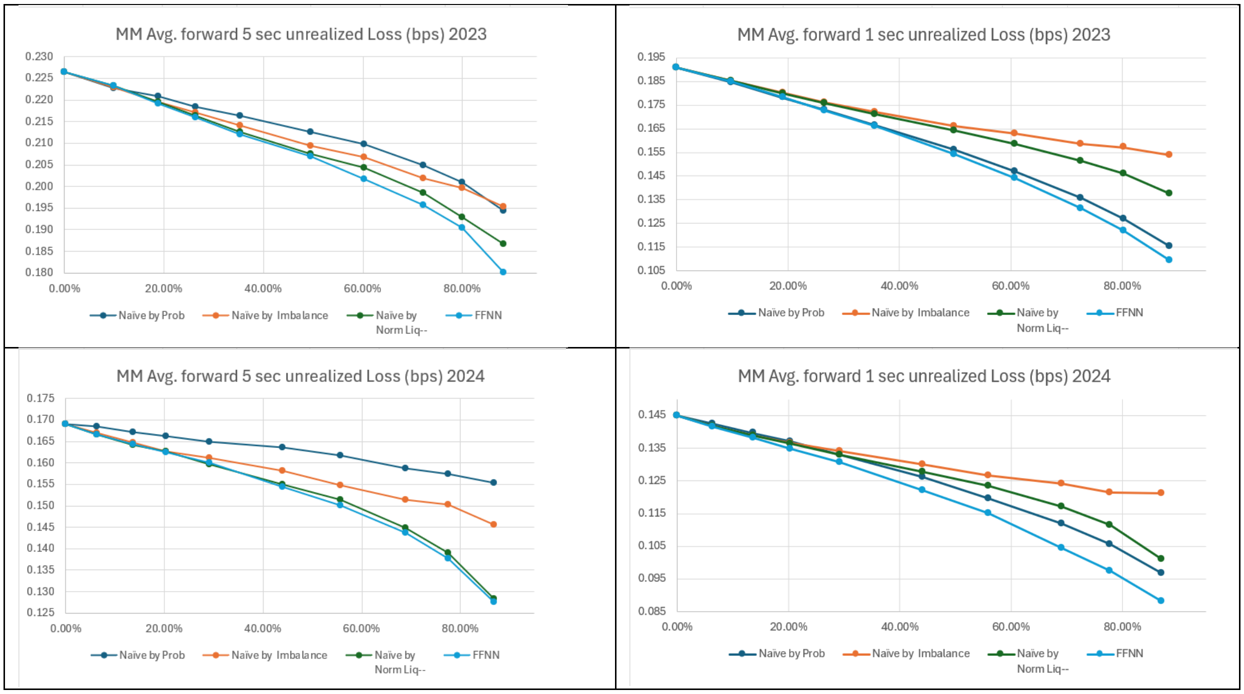


Figure 12. Forward average loss at different cancellation rates of the FFNN scoring function vs naïve algorithms utilizing three different scoring factors. FFNN out of sample years 2021-2024.

Observations:

- The algorithm utilizing FFNN learnt scoring function outperforms all naïve models on every out-of-sample year: 2021-2024.
- The FFNN dominance is much sharper on 1 second forward loss (see loss attribution chart as well).
- That's true in both projections: (i) a lower unrealized loss for the same cancellation rate; and (ii) lower cancellation rate (and higher trading turnover) for the same target loss level.

Conclusion

In this study we analysed the short-term price dynamics of the CME ES front contracts at the moments of observed TOB quote imbalances. Our findings confirm that TOB imbalance could serve as an effective predictor of near-term market movement direction, with statistical significance validated over a 5-second post-event horizon.

We evaluated a few other factors as predictors of near-term market movement direction, alone or in combination with imbalance factor. One of them is probability of the market move in the direction implied by imbalance by the end of 5-sec post event horizon. To quantify such a probability, we employed a driftless random walk model. With realized volatility estimated from one-second log returns of the weighted mid-price process, and a price barrier calculated as a distance to the nearest weighted mid-price that ensures one tick change of the TOB prices in the direction of imbalance, the RW model probability is matched well with probability estimated directly from the sample data.

With thousands of imbalance events occurring per session, a typical decision for the market maker or any other execution algorithm, which is keeping a quote or order on the illiquid side of the TOB is to decide whether to continue to quote the market or cancel the quote. We frame this as an optimization problem: find a scoring function that minimizes the expected unrealized forward loss (1- and 5-second horizons) under a simple decision policy across various cancellation rates.

Moving beyond single-factor heuristic algorithms, we introduce a "Percentile Optimization Framework" to train a multi-factor scoring function parameterized by a Feed-Forward Neural Network (FFNN).

This framework does not merely predict risk but directly optimizes the outcome of the decision policy itself. By minimizing a loss function that is a sum of average losses across multiple cancellation rates, the model learns a robust scoring function that performs well under both aggressive and conservative operational constraints (target cancellation rate), ensuring the learned strategy is directly tied to forward P&L/loss.

The empirical results are decisive. When trained on data from 2018-2020 and evaluated on a multi-year out-of-sample period (2021-2024), the FFNN-based algorithm demonstrated consistent outperformance over all single-factor benchmarks. This pattern was observed across every out-of-sample year and for both forward time horizons.

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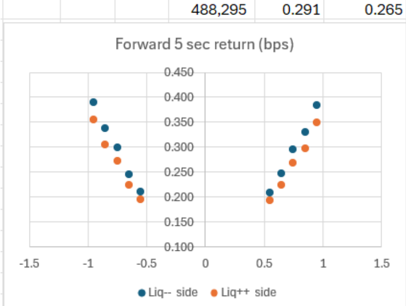
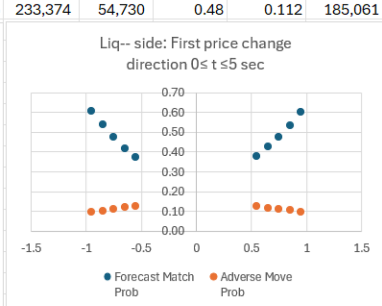
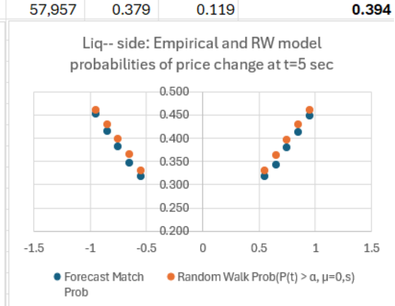
Appendix 1. Imbalance bucketing analysis for 2018-2024

Year 2018 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: First price change direction within 0<=t<=5 sec from event time				**Liq--** side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	35,641	0.384	0.349	21,439	3,395	0.60	0.095	15,993	3,890	0.449	0.109	0.460
0.8	0.9	0.85	56,314	0.330	0.298	30,090	5,958	0.53	0.106	23,228	6,452	0.412	0.115	0.429
0.7	0.8	0.75	56,288	0.294	0.267	26,732	6,305	0.47	0.112	21,380	6,625	0.380	0.118	0.396
0.6	0.7	0.65	50,878	0.246	0.224	21,580	6,015	0.42	0.118	17,457	6,328	0.343	0.124	0.364
0.5	0.6	0.55	46,257	0.209	0.192	17,418	5,868	0.38	0.127	14,677	5,889	0.317	0.127	0.330
-0.5	-0.6	-0.55	45,579	0.210	0.195	17,088	5,726	0.37	0.126	14,484	5,799	0.318	0.127	0.330
-0.6	-0.7	-0.65	50,848	0.246	0.225	21,300	6,197	0.42	0.122	17,656	6,371	0.347	0.125	0.365
-0.7	-0.8	-0.75	55,651	0.298	0.271	26,525	6,160	0.48	0.111	21,200	6,463	0.381	0.116	0.397
-0.8	-0.9	-0.85	55,521	0.337	0.305	29,847	5,730	0.54	0.103	23,018	6,303	0.415	0.114	0.430
-0.9	-1	-0.95	35,318	0.389	0.356	21,355	3,376	0.60	0.096	15,968	3,837	0.452	0.109	0.461
			488,295	0.291	0.265	233,374	54,730	0.48	0.112	185,061	57,957	0.379	0.119	**0.394**



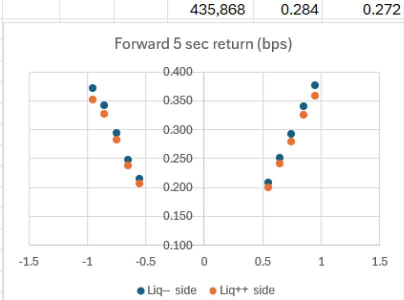


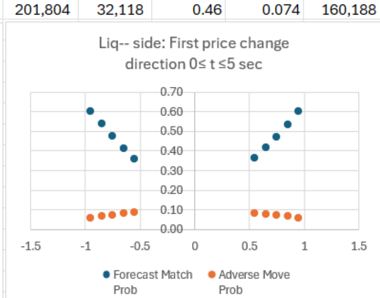


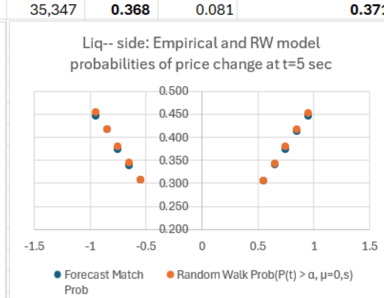
RMSE = 0.015 (Forecast Match Prob vs RW Prob Estimate)

Year 2019 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: First price change direction within 0<=t<=5 sec from event time				**Liq--** side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	25,321	0.376	0.359	15,193	1,513	0.60	0.060	11,295	1,776	0.446	0.070	0.453
0.8	0.9	0.85	45,339	0.340	0.325	24,198	2,959	0.53	0.065	18,664	3,300	0.412	0.073	0.416
0.7	0.8	0.75	50,345	0.291	0.279	23,627	3,705	0.47	0.074	18,815	4,086	0.374	0.081	0.379
0.6	0.7	0.65	49,410	0.250	0.240	20,657	3,892	0.42	0.079	16,784	4,222	0.340	0.085	0.342
0.5	0.6	0.55	48,361	0.208	0.200	17,605	3,994	0.36	0.083	14,740	4,393	0.305	0.091	0.305
-0.5	-0.6	-0.55	47,518	0.214	0.206	17,109	4,088	0.36	0.086	14,569	4,228	0.307	0.089	0.307
-0.6	-0.7	-0.65	49,372	0.247	0.237	20,393	3,974	0.41	0.080	16,692	4,218	0.338	0.085	0.344
-0.7	-0.8	-0.75	50,353	0.293	0.282	23,819	3,547	0.47	0.070	18,780	3,970	0.373	0.079	0.380
-0.8	-0.9	-0.85	45,152	0.342	0.327	24,322	3,015	0.54	0.067	18,830	3,422	0.417	0.076	0.417
-0.9	-1	-0.95	24,697	0.371	0.351	14,881	1,431	0.60	0.058	11,019	1,732	0.446	0.070	0.453
			435,868	0.284	0.272	201,804	32,118	0.46	0.074	160,188	35,347	**0.368**	0.081	**0.371**



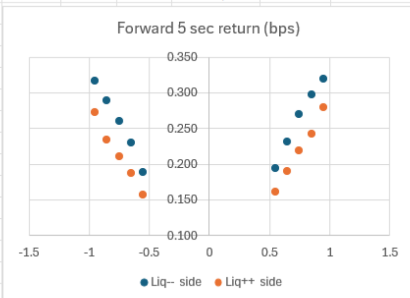


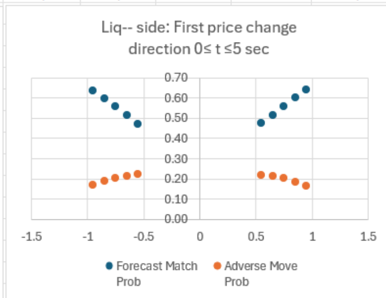


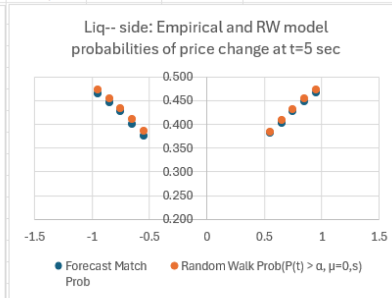
RMSE = 0.0049 (Forecast Match Prob vs RW Prob Estimate)

Year 2020 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: First price change direction within 0<= t <=5 sec from event time				Liq-- side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	36,854	0.319	0.279	23,583	6,078	0.64	0.165	17,178	6,553	0.466	0.178	0.473
0.8	0.9	0.85	71,119	0.297	0.242	42,686	13,035	0.60	0.183	31,862	13,469	0.448	0.189	0.454
0.7	0.8	0.75	74,011	0.269	0.219	41,253	14,915	0.56	0.202	31,646	14,888	0.428	0.201	0.432
0.6	0.7	0.65	68,598	0.231	0.189	35,301	14,527	0.51	0.212	27,625	14,306	0.403	0.209	0.409
0.5	0.6	0.55	61,415	0.194	0.161	29,289	13,419	0.48	0.218	23,390	13,116	0.381	0.214	0.384
-0.5	-0.6	-0.55	60,269	0.189	0.157	28,387	13,381	0.47	0.222	22,620	12,998	0.375	0.216	0.386
-0.6	-0.7	-0.65	68,293	0.230	0.187	35,122	14,402	0.51	0.211	27,380	14,210	0.401	0.208	0.410
-0.7	-0.8	-0.75	72,756	0.259	0.210	40,624	14,765	0.56	0.203	31,004	14,822	0.426	0.204	0.433
-0.8	-0.9	-0.85	69,356	0.288	0.233	41,445	12,974	0.60	0.187	30,960	13,437	0.446	0.194	0.455
-0.9	-1	-0.95	36,143	0.316	0.273	23,004	6,069	0.64	0.168	16,751	6,536	0.463	0.181	0.473
			618,814	0.255	0.210	340,694	123,565	0.55	0.200	260,416	124,335	0.421	0.201	**0.428**



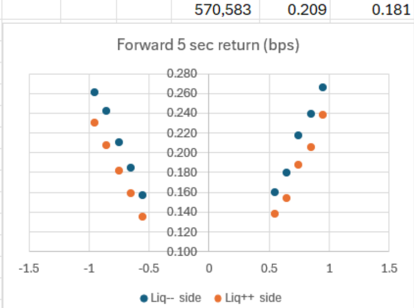


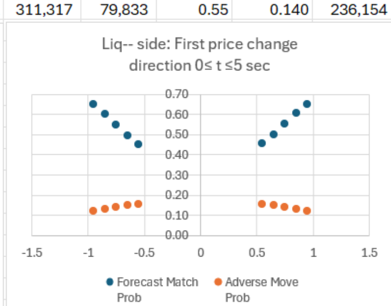


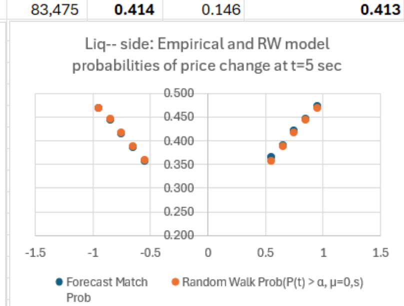
RMSE = 0.0073 (Forecast Match Prob vs RW Prob Estimate)

Year 2021 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: First price change direction within 0<= t <=5 sec from event time				Liq-- side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	42,048	0.266	0.237	27,253	5,033	0.65	0.120	19,879	5,557	0.473	0.132	0.467
0.8	0.9	0.85	64,145	0.238	0.205	38,716	8,389	0.60	0.131	28,556	9,115	0.445	0.142	0.444
0.7	0.8	0.75	63,639	0.217	0.187	35,133	9,023	0.55	0.142	26,837	9,443	0.422	0.148	0.417
0.6	0.7	0.65	61,354	0.179	0.153	30,595	9,109	0.50	0.148	23,889	9,283	0.389	0.151	0.387
0.5	0.6	0.55	56,832	0.160	0.138	25,931	8,722	0.46	0.153	20,759	8,702	0.365	0.153	0.358
-0.5	-0.6	-0.55	56,170	0.156	0.135	25,237	8,556	0.45	0.152	20,034	8,626	0.357	0.154	0.359
-0.6	-0.7	-0.65	59,804	0.185	0.159	29,624	8,856	0.50	0.148	23,123	9,005	0.387	0.151	0.388
-0.7	-0.8	-0.75	62,422	0.210	0.182	34,262	8,882	0.55	0.142	25,857	9,373	0.414	0.150	0.417
-0.8	-0.9	-0.85	63,109	0.242	0.208	37,950	8,367	0.60	0.133	27,970	8,886	0.443	0.141	0.445
-0.9	-1	-0.95	41,060	0.261	0.230	26,616	4,896	0.65	0.119	19,250	5,485	0.469	0.134	0.468
			570,583	0.209	0.181	311,317	79,833	0.55	0.140	236,154	83,475	**0.414**	0.146	**0.413**



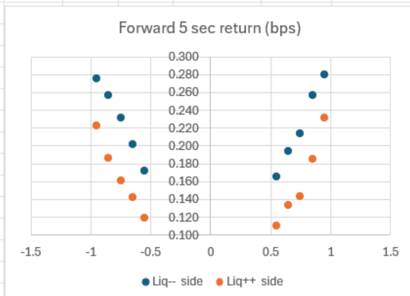


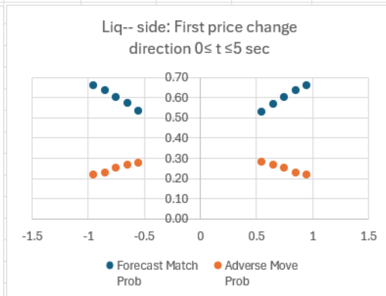


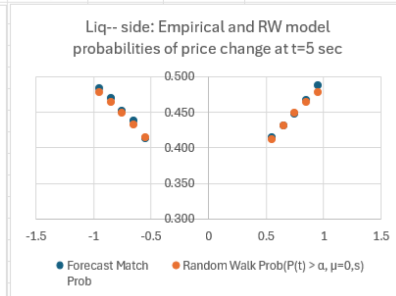
RMSE = 0.0038 (Forecast Match Prob vs RW Prob Estimate)

Year 2022 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: First price change direction within 0<=t<=5 sec from event time				Liq-- side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	27,724	0.279	0.231	18,305	6,060	0.66	0.219	13,508	6,384	0.487	0.230	0.478
0.8	0.9	0.85	69,935	0.257	0.185	44,350	15,967	0.63	0.228	32,666	16,245	0.467	0.232	0.464
0.7	0.8	0.75	70,686	0.214	0.144	42,585	17,953	0.60	0.254	31,610	17,891	0.447	0.253	0.448
0.6	0.7	0.65	64,575	0.194	0.134	36,547	17,205	0.57	0.266	27,817	16,754	0.431	0.259	0.430
0.5	0.6	0.55	58,254	0.165	0.110	30,874	16,368	0.53	0.281	24,096	15,652	0.414	0.269	0.412
-0.5	-0.6	-0.55	57,546	0.172	0.119	30,688	15,936	0.53	0.277	23,791	15,350	0.413	0.267	0.414
-0.6	-0.7	-0.65	63,857	0.202	0.142	36,463	16,921	0.57	0.265	27,932	16,369	0.437	0.256	0.432
-0.7	-0.8	-0.75	69,754	0.231	0.160	42,017	17,595	0.60	0.252	31,509	17,341	0.452	0.249	0.449
-0.8	-0.9	-0.85	68,959	0.257	0.186	43,859	15,722	0.64	0.228	32,361	15,919	0.469	0.231	0.464
-0.9	-1	-0.95	27,047	0.275	0.223	17,811	5,916	0.66	0.219	13,064	6,285	0.483	0.232	0.478
			578,337	0.220	0.156	343,499	145,643	0.59	0.252	258,354	144,190	0.447	0.249	**0.444**



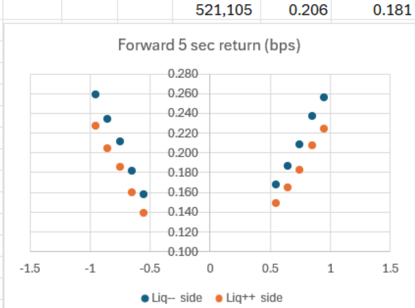


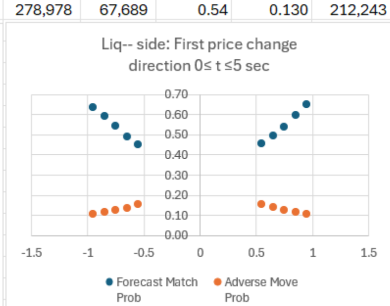


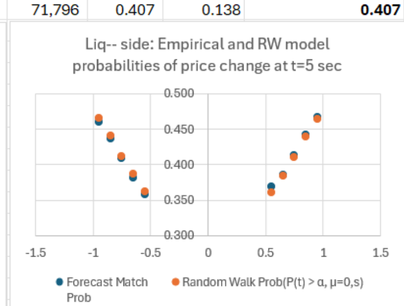
RMSE = 0.0045 (Forecast Match Prob vs RW Prob Estimate)

Year 2023 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: First price change direction within 0<=t<=5 sec from event time				Liq-- side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	35,023	0.255	0.224	22,718	3,715	0.65	0.106	16,331	4,154	0.466	0.119	0.464
0.8	0.9	0.85	54,288	0.237	0.207	32,440	6,229	0.60	0.115	23,984	6,864	0.442	0.126	0.439
0.7	0.8	0.75	58,004	0.208	0.183	31,175	7,409	0.54	0.128	23,926	7,863	0.412	0.136	0.410
0.6	0.7	0.65	58,186	0.186	0.165	28,704	8,043	0.49	0.138	22,438	8,432	0.386	0.145	0.384
0.5	0.6	0.55	55,688	0.167	0.149	25,323	8,612	0.45	0.155	20,530	8,541	0.369	0.153	0.360
-0.5	-0.6	-0.55	55,389	0.158	0.139	24,954	8,563	0.45	0.155	19,806	8,717	0.358	0.157	0.362
-0.6	-0.7	-0.65	58,604	0.181	0.160	28,588	8,034	0.49	0.137	22,299	8,505	0.381	0.145	0.386
-0.7	-0.8	-0.75	57,415	0.211	0.185	31,193	7,079	0.54	0.123	23,492	7,660	0.409	0.133	0.411
-0.8	-0.9	-0.85	53,743	0.234	0.204	31,724	6,325	0.59	0.118	23,462	6,942	0.437	0.129	0.440
-0.9	-1	-0.95	34,765	0.259	0.227	22,159	3,680	0.64	0.106	15,975	4,118	0.460	0.118	0.465
			521,105	0.206	0.181	278,978	67,689	0.54	0.130	212,243	71,796	0.407	0.138	**0.407**



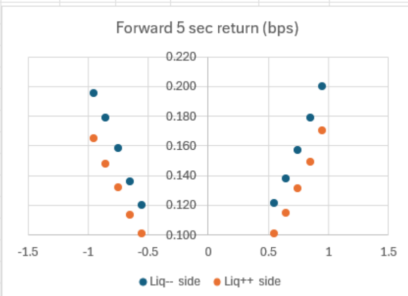


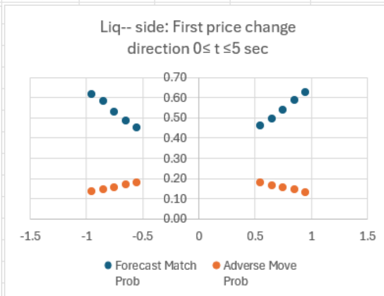


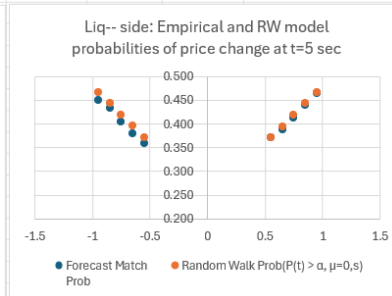
RMSE = 0.0043 (Forecast Match Prob vs RW Prob Estimate)

Year 2024 Results

Imbalance buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: First price change direction within 0<= t <=5 sec from event time				Liq-- side: Forward price change direction at t = 5 sec from event time				Random Walk Prob(P(t) > α, μ=0,σ)
From	To	Mid-range	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob	of price P(t)to finish above α at t = 5 (sec) Liq-- side
0.9	1	0.95	21,131	0.200	0.170	13,259	2,749	0.63	0.130	9,810	2,975	0.464	0.141	0.466
0.8	0.9	0.85	45,210	0.179	0.149	26,586	6,485	0.59	0.143	19,909	6,719	0.440	0.149	0.444
0.7	0.8	0.75	55,402	0.157	0.131	29,903	8,441	0.54	0.152	22,882	8,716	0.413	0.157	0.418
0.6	0.7	0.65	57,284	0.138	0.115	28,233	9,413	0.49	0.164	22,190	9,606	0.387	0.168	0.394
0.5	0.6	0.55	55,442	0.121	0.101	25,497	9,862	0.46	0.178	20,553	9,824	0.371	0.177	0.371
-0.5	-0.6	-0.55	54,749	0.120	0.101	24,609	9,851	0.45	0.180	19,703	9,767	0.360	0.178	0.372
-0.6	-0.7	-0.65	56,544	0.136	0.113	27,289	9,612	0.48	0.170	21,484	9,790	0.380	0.173	0.395
-0.7	-0.8	-0.75	54,523	0.158	0.132	28,706	8,501	0.53	0.156	22,036	8,758	0.404	0.161	0.419
-0.8	-0.9	-0.85	43,925	0.179	0.148	25,497	6,397	0.58	0.146	19,009	6,705	0.433	0.153	0.445
-0.9	-1	-0.95	20,586	0.195	0.165	12,684	2,772	0.62	0.135	9,264	3,004	0.450	0.146	0.467
			464,796	0.152	0.126	242,263	74,083	0.52	0.159	186,840	75,864	0.402	0.163	**0.411**





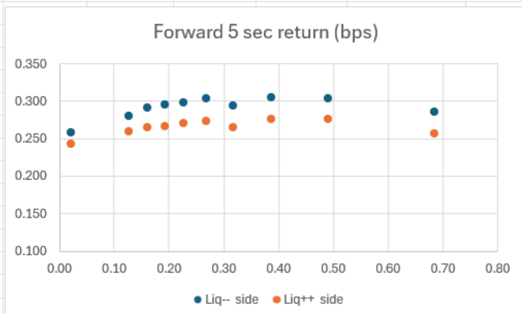


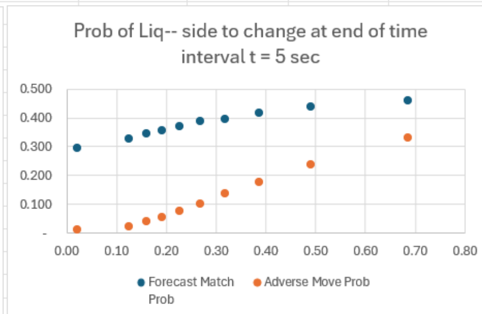
RMSE = 0.0105 (Forecast Match Prob vs RW Prob Estimate)

Appendix 2. Volatility factor analysis 2018-2024

Year 2018 Results

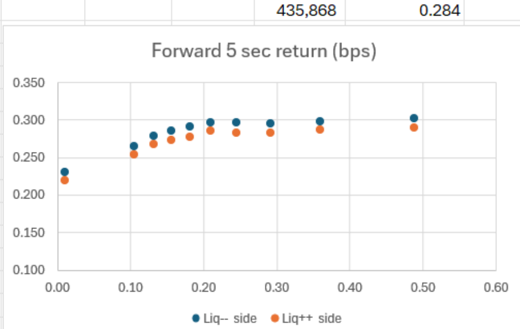
Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.69	8.78	0.90	48,830	0.286	0.257	22,480	16,200	0.460	0.332
0.49	0.69	0.80	48,829	0.303	0.276	21,406	11,544	0.438	0.236
0.39	0.49	0.70	48,830	0.305	0.276	20,318	8,661	0.416	0.177
0.32	0.39	0.60	48,829	0.294	0.265	19,254	6,704	0.394	0.137
0.27	0.32	0.50	48,830	0.303	0.273	18,861	4,899	0.386	0.100
0.23	0.27	0.40	48,829	0.298	0.270	18,147	3,706	0.372	0.076
0.19	0.23	0.30	48,829	0.295	0.267	17,445	2,657	0.357	0.054
0.16	0.19	0.20	48,830	0.290	0.265	16,825	1,874	0.345	0.038
0.13	0.16	0.10	48,829	0.280	0.259	15,894	1,124	0.326	0.023
0.02	0.13	0.00	48,830	0.257	0.242	14,432	588	0.296	0.012
			488,295	0.291	0.265	185,062	57,957	0.379	0.119

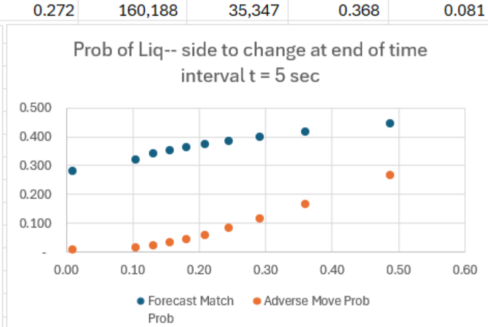




Year 2019 Results

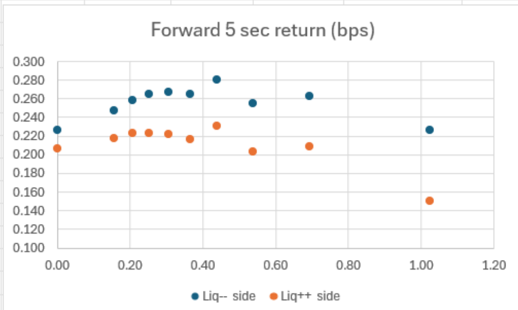
Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.49	5.90	0.90	43,587	0.302	0.290	19,362	11,601	0.444	0.266
0.36	0.49	0.80	43,587	0.298	0.286	18,149	7,229	0.416	0.166
0.29	0.36	0.70	43,587	0.295	0.283	17,412	5,040	0.399	0.116
0.24	0.29	0.60	43,586	0.296	0.283	16,786	3,618	0.385	0.083
0.21	0.24	0.50	43,587	0.297	0.285	16,300	2,571	0.374	0.059
0.18	0.21	0.40	43,587	0.291	0.278	15,766	1,950	0.362	0.045
0.16	0.18	0.30	43,586	0.285	0.273	15,338	1,457	0.352	0.033
0.13	0.16	0.20	43,587	0.279	0.267	14,828	951	0.340	0.022
0.10	0.13	0.10	43,587	0.265	0.254	13,998	608	0.321	0.014
0.01	0.10	0.00	43,587	0.230	0.220	12,249	322	0.281	0.007
			435,868	0.284	0.272	160,188	35,347	0.368	0.081

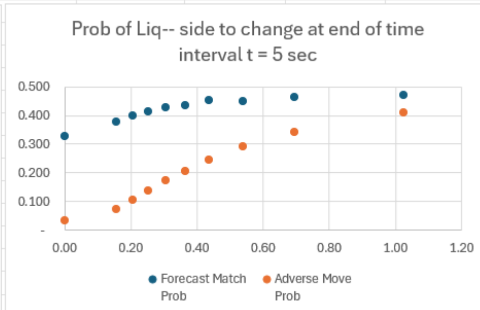




Year 2020 Results

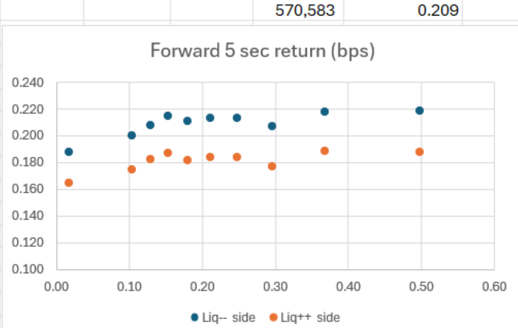
Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
1.02	10.95	0.90	61,882	0.226	0.151	29,181	25,399	0.472	0.410
0.69	1.02	0.80	61,881	0.263	0.209	28,613	21,093	0.462	0.341
0.54	0.69	0.70	61,881	0.255	0.203	27,755	18,036	0.449	0.291
0.44	0.54	0.60	61,882	0.280	0.231	27,952	15,063	0.452	0.243
0.36	0.44	0.50	61,881	0.264	0.217	26,861	12,714	0.434	0.205
0.31	0.36	0.40	61,881	0.267	0.222	26,351	10,607	0.426	0.171
0.25	0.31	0.30	61,882	0.264	0.223	25,576	8,523	0.413	0.138
0.21	0.25	0.20	61,881	0.258	0.222	24,606	6,460	0.398	0.104
0.16	0.21	0.10	61,881	0.248	0.217	23,239	4,444	0.376	0.072
0.00	0.16	0.00	61,882	0.226	0.206	20,282	1,996	0.328	0.032
			618,814	0.255	0.210	260,416	124,335	0.421	0.201

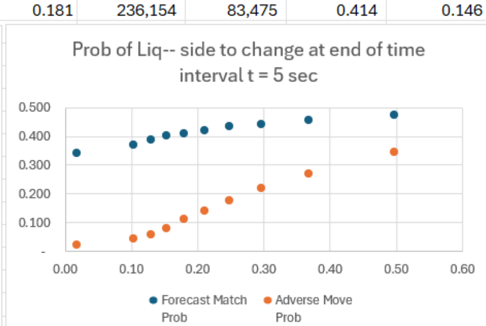




Year 2021 Results

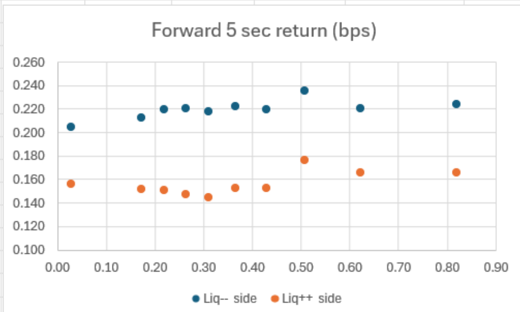
Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.50	5.33	0.90	57,059	0.219	0.188	27,110	19,635	0.475	0.344
0.37	0.50	0.80	57,058	0.218	0.189	26,019	15,307	0.456	0.268
0.30	0.37	0.70	57,058	0.207	0.177	25,259	12,606	0.443	0.221
0.25	0.30	0.60	57,058	0.213	0.183	24,743	10,036	0.434	0.176
0.21	0.25	0.50	57,059	0.213	0.184	23,972	7,961	0.420	0.140
0.18	0.21	0.40	57,058	0.211	0.182	23,432	6,380	0.411	0.112
0.15	0.18	0.30	57,058	0.214	0.187	22,926	4,628	0.402	0.081
0.13	0.15	0.20	57,058	0.208	0.182	22,114	3,320	0.388	0.058
0.10	0.13	0.10	57,058	0.200	0.174	21,152	2,398	0.371	0.042
0.02	0.10	0.00	57,059	0.188	0.165	19,427	1,204	0.340	0.021
			570,583	0.209	0.181	236,154	83,475	0.414	0.146

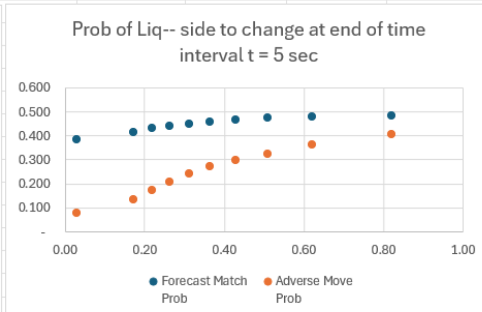




Year 2022 Results

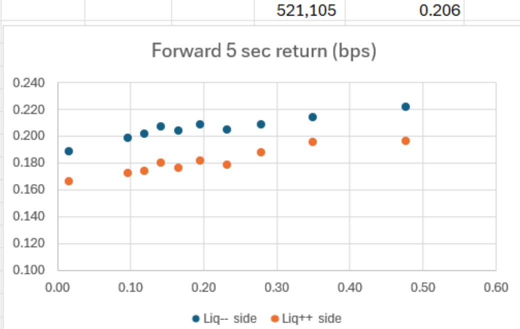
Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.82	9.36	0.90	57,834	0.224	0.166	27,985	23,390	0.484	0.404
0.62	0.82	0.80	57,834	0.221	0.165	27,667	20,902	0.478	0.361
0.51	0.62	0.70	57,833	0.236	0.176	27,326	18,748	0.472	0.324
0.43	0.51	0.60	57,834	0.219	0.153	26,850	17,287	0.464	0.299
0.36	0.43	0.50	57,834	0.222	0.153	26,369	15,634	0.456	0.270
0.31	0.36	0.40	57,833	0.218	0.145	25,912	14,007	0.448	0.242
0.26	0.31	0.30	57,834	0.220	0.148	25,407	11,879	0.439	0.205
0.22	0.26	0.20	57,833	0.220	0.151	24,915	9,920	0.431	0.172
0.17	0.22	0.10	57,834	0.212	0.152	23,834	7,823	0.412	0.135
0.03	0.17	0.00	57,834	0.205	0.156	22,090	4,600	0.382	0.080
			578,337	0.220	0.156	258,355	144,190	0.447	0.249

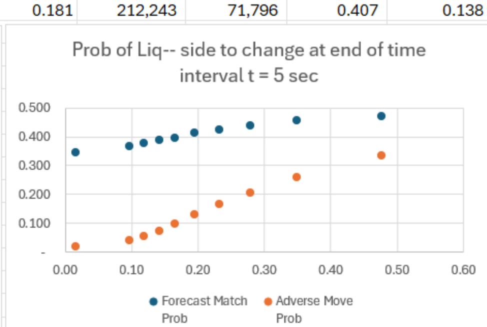




Year 2023 Results

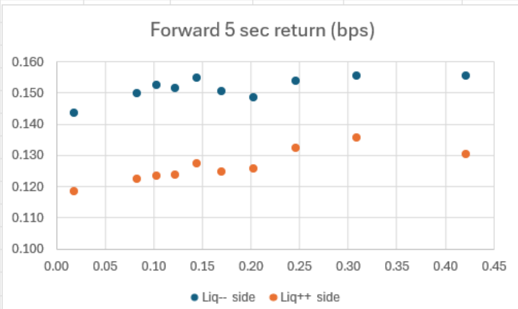
Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.48	8.15	0.90	52,111	0.222	0.196	24,602	17,505	0.472	0.336
0.35	0.48	0.80	52,110	0.214	0.195	23,712	13,458	0.455	0.258
0.28	0.35	0.70	52,111	0.208	0.187	22,774	10,749	0.437	0.206
0.23	0.28	0.60	52,110	0.204	0.179	22,089	8,577	0.424	0.165
0.20	0.23	0.50	52,111	0.208	0.181	21,496	6,707	0.413	0.129
0.17	0.20	0.40	52,110	0.204	0.176	20,564	5,173	0.395	0.099
0.14	0.17	0.30	52,110	0.207	0.180	20,288	3,742	0.389	0.072
0.12	0.14	0.20	52,111	0.201	0.174	19,649	2,855	0.377	0.055
0.10	0.12	0.10	52,110	0.199	0.172	19,146	2,002	0.367	0.038
0.02	0.10	0.00	52,111	0.188	0.166	17,923	1,028	0.344	0.020
			521,105	0.206	0.181	212,243	71,796	0.407	0.138

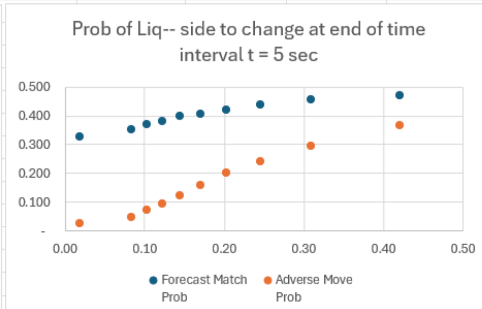




Year 2024 Results

Volatility Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.42	8.20	0.90	46,480	0.155	0.130	21,821	17,089	0.469	0.368
0.31	0.42	0.80	46,480	0.155	0.136	21,210	13,772	0.456	0.296
0.25	0.31	0.70	46,479	0.154	0.132	20,334	11,242	0.437	0.242
0.20	0.25	0.60	46,480	0.149	0.126	19,488	9,366	0.419	0.202
0.17	0.20	0.50	46,479	0.150	0.125	18,925	7,441	0.407	0.160
0.14	0.17	0.40	46,480	0.155	0.127	18,534	5,710	0.399	0.123
0.12	0.14	0.30	46,479	0.151	0.124	17,691	4,424	0.381	0.095
0.10	0.12	0.20	46,480	0.152	0.123	17,160	3,295	0.369	0.071
0.08	0.10	0.10	46,479	0.150	0.122	16,446	2,270	0.354	0.049
0.02	0.08	0.00	46,480	0.143	0.118	15,233	1,255	0.328	0.027
			464,796	0.152	0.126	186,842	75,864	0.402	0.163



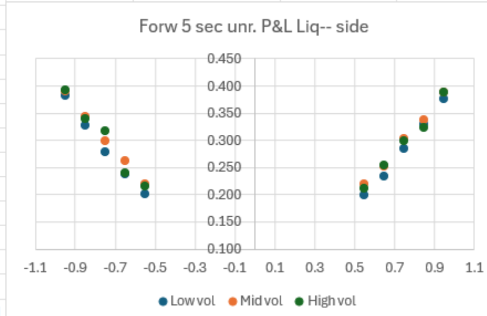


Appendix 3. Two-dimensional bucket analysis (imbalance x volatility factors) 2018-2024

Year 2018 Results

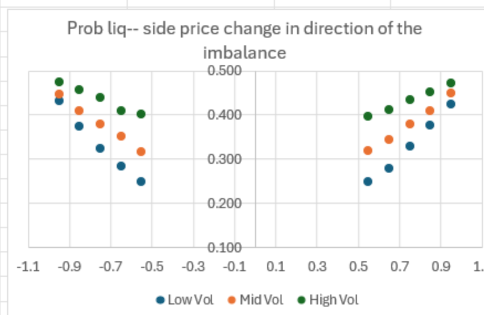
Forw 5 sec unr. P&L Liq-- side

		Volatility	0.203	0.361	8.780	
Imbalance Buckets		**Buckets**	0.021	0.203	0.361	
0.9	1	0.95	0.376	0.387	0.387	0.384
0.8	0.9	0.85	0.329	0.338	0.324	0.330
0.7	0.8	0.75	0.284	0.302	0.298	0.294
0.6	0.7	0.65	0.233	0.252	0.253	0.246
0.5	0.6	0.55	0.198	0.219	0.211	0.209
-0.5	-0.6	-0.55	0.200	0.218	0.214	0.210
-0.6	-0.7	-0.65	0.237	0.261	0.239	0.246
-0.7	-0.8	-0.75	0.278	0.298	0.318	0.298
-0.8	-0.9	-0.85	0.327	0.343	0.340	0.337
-0.9	-1	-0.95	0.383	0.391	0.393	0.389
			0.278	0.298	0.297	**0.291**



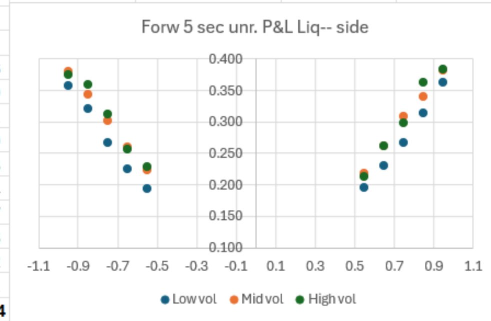
Prob of Liq-- price change in direction matching imbalance

		Volatility	0.203	0.361	8.780	
Imbalance Buckets		**buckets**	0.021	0.203	0.361	
0.9	1	0.95	0.424	0.448	0.472	0.449
0.8	0.9	0.85	0.376	0.409	0.451	0.412
0.7	0.8	0.75	0.329	0.378	0.433	0.380
0.6	0.7	0.65	0.279	0.344	0.410	0.343
0.5	0.6	0.55	0.249	0.319	0.397	0.317
-0.5	-0.6	-0.55	0.248	0.316	0.401	0.318
-0.6	-0.7	-0.65	0.283	0.351	0.409	0.347
-0.7	-0.8	-0.75	0.324	0.378	0.440	0.381
-0.8	-0.9	-0.85	0.374	0.409	0.457	0.415
-0.9	-1	-0.95	0.431	0.447	0.474	0.452
			0.325	0.377	0.434	**0.379**

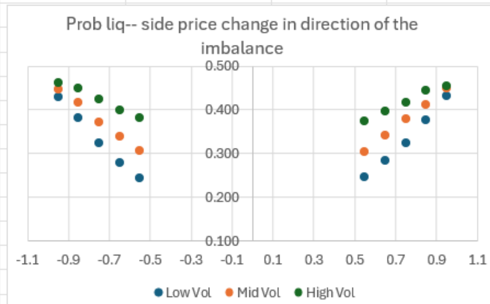


Year 2019 Results

Forw 5 sec unr. P&L Liq-- side						
		Volatility	0.164	0.274	5.899	
Imbalance Buckets		Buckets	0.011	0.164	0.274	
0.9	1	0.95	0.362	0.382	0.383	0.376
0.8	0.9	0.85	0.314	0.339	0.363	0.340
0.7	0.8	0.75	0.267	0.308	0.299	0.291
0.6	0.7	0.65	0.230	0.261	0.261	0.250
0.5	0.6	0.55	0.195	0.218	0.212	0.208
-0.5	-0.6	-0.55	0.193	0.223	0.228	0.214
-0.6	-0.7	-0.65	0.225	0.260	0.256	0.247
-0.7	-0.8	-0.75	0.266	0.302	0.311	0.293
-0.8	-0.9	-0.85	0.320	0.344	0.359	0.342
-0.9	-1	-0.95	0.357	0.380	0.374	0.371
			0.260	0.293	0.298	**0.284**

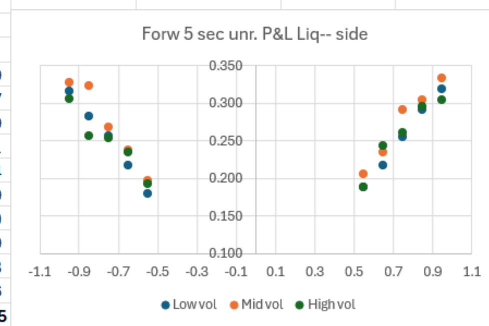


Prob of Liq-- price change in direction matching imbalance						
		Volatility	0.164	0.274	5.899	
Imbalance Buckets		buckets	0.011	0.164	0.274	
0.9	1	0.95	0.431	0.450	0.455	0.446
0.8	0.9	0.85	0.377	0.411	0.445	0.412
0.7	0.8	0.75	0.324	0.379	0.417	0.374
0.6	0.7	0.65	0.283	0.342	0.397	0.340
0.5	0.6	0.55	0.245	0.304	0.375	0.305
-0.5	-0.6	-0.55	0.243	0.306	0.381	0.307
-0.6	-0.7	-0.65	0.278	0.339	0.398	0.338
-0.7	-0.8	-0.75	0.323	0.371	0.424	0.373
-0.8	-0.9	-0.85	0.381	0.417	0.450	0.417
-0.9	-1	-0.95	0.428	0.448	0.461	0.446
			0.317	0.368	0.417	**0.368**

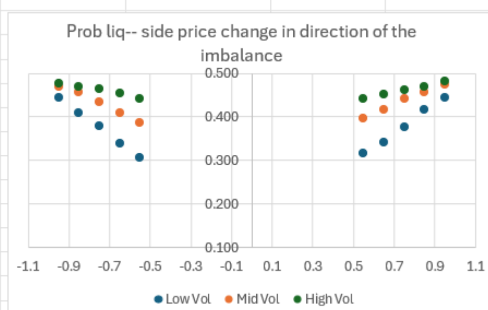


Year 2020 Results

Forw 5 sec unr. P&L Liq-- side						
		Volatility	0.270	0.501	10.950	
Imbalance Buckets		Buckets	0.000	0.270	0.501	
0.9	1	0.95	0.318	0.333	0.305	0.319
0.8	0.9	0.85	0.291	0.304	0.295	0.297
0.7	0.8	0.75	0.255	0.291	0.261	0.269
0.6	0.7	0.65	0.216	0.235	0.243	0.231
0.5	0.6	0.55	0.187	0.206	0.188	0.194
-0.5	-0.6	-0.55	0.179	0.196	0.193	0.189
-0.6	-0.7	-0.65	0.217	0.238	0.235	0.230
-0.7	-0.8	-0.75	0.257	0.267	0.254	0.259
-0.8	-0.9	-0.85	0.283	0.323	0.257	0.288
-0.9	-1	-0.95	0.315	0.327	0.306	0.316
			0.246	0.270	0.250	**0.255**



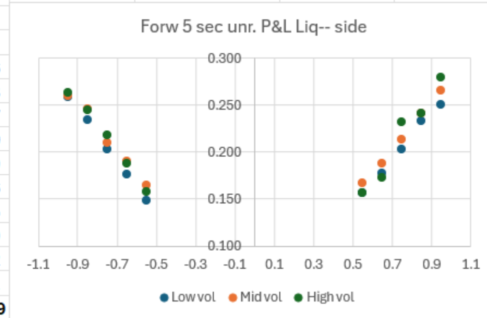
Prob of Liq-- price change in direction matching imbalance						
		Volatility	0.270	0.501	10.950	
Imbalance Buckets		buckets	0.000	0.270	0.501	
0.9	1	0.95	0.444	0.473	0.481	0.466
0.8	0.9	0.85	0.417	0.456	0.469	0.448
0.7	0.8	0.75	0.377	0.442	0.461	0.428
0.6	0.7	0.65	0.341	0.417	0.451	0.403
0.5	0.6	0.55	0.315	0.396	0.440	0.381
-0.5	-0.6	-0.55	0.305	0.386	0.442	0.375
-0.6	-0.7	-0.65	0.339	0.410	0.454	0.401
-0.7	-0.8	-0.75	0.379	0.433	0.463	0.426
-0.8	-0.9	-0.85	0.410	0.457	0.470	0.446
-0.9	-1	-0.95	0.444	0.468	0.478	0.463
			0.371	0.432	0.460	**0.421**



Year 2021 Results

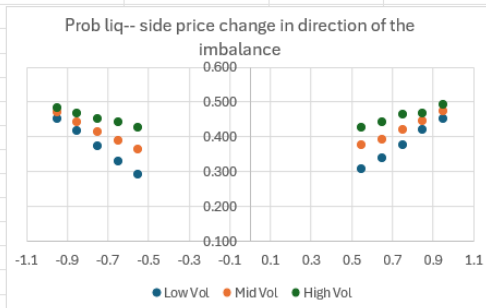
Forw 5 sec unr. P&L Liq-- side

		Volatility	0.162	0.278	5.329	
Imbalance Buckets		**Buckets**	0.018	0.162	0.278	
0.9	1	0.95	0.251	0.266	0.280	0.266
0.8	0.9	0.85	0.233	0.241	0.241	0.238
0.7	0.8	0.75	0.203	0.214	0.232	0.217
0.6	0.7	0.65	0.178	0.188	0.173	0.179
0.5	0.6	0.55	0.157	0.167	0.156	0.160
-0.5	-0.6	-0.55	0.148	0.165	0.157	0.156
-0.6	-0.7	-0.65	0.176	0.191	0.188	0.185
-0.7	-0.8	-0.75	0.203	0.210	0.218	0.210
-0.8	-0.9	-0.85	0.235	0.246	0.244	0.242
-0.9	-1	-0.95	0.258	0.260	0.263	0.261
			0.200	0.213	0.215	**0.209**



Prob of Liq-- price change in direction matching imbalance

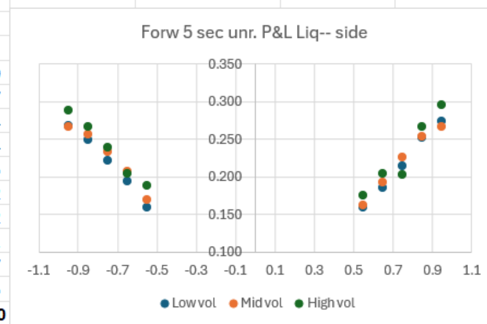
		Volatility	0.162	0.278	5.329	
Imbalance Buckets		**buckets**	0.018	0.162	0.278	
0.9	1	0.95	0.451	0.473	0.493	0.473
0.8	0.9	0.85	0.419	0.447	0.467	0.445
0.7	0.8	0.75	0.377	0.420	0.464	0.422
0.6	0.7	0.65	0.337	0.394	0.441	0.389
0.5	0.6	0.55	0.307	0.376	0.427	0.365
-0.5	-0.6	-0.55	0.292	0.365	0.427	0.357
-0.6	-0.7	-0.65	0.331	0.390	0.443	0.387
-0.7	-0.8	-0.75	0.373	0.413	0.453	0.414
-0.8	-0.9	-0.85	0.417	0.442	0.467	0.443
-0.9	-1	-0.95	0.452	0.469	0.484	0.469
			0.369	0.417	0.456	**0.414**



Year 2022 Results

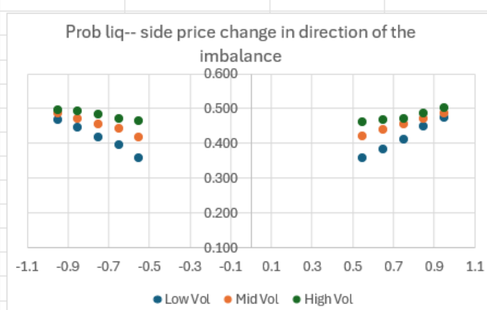
Forw 5 sec unr. P&L Liq-- side

		Volatility	0.279	0.479	9.359	
Imbalance Buckets		**Buckets**	0.029	0.279	0.479	
0.9	1	0.95	0.273	0.267	0.295	0.279
0.8	0.9	0.85	0.252	0.254	0.266	0.257
0.7	0.8	0.75	0.214	0.225	0.203	0.214
0.6	0.7	0.65	0.186	0.192	0.205	0.194
0.5	0.6	0.55	0.158	0.162	0.175	0.165
-0.5	-0.6	-0.55	0.158	0.169	0.189	0.172
-0.6	-0.7	-0.65	0.193	0.207	0.204	0.202
-0.7	-0.8	-0.75	0.221	0.233	0.238	0.231
-0.8	-0.9	-0.85	0.249	0.256	0.266	0.257
-0.9	-1	-0.95	0.268	0.266	0.288	0.275
			0.213	0.219	0.227	**0.220**



Prob of Liq-- price change in direction matching imbalance

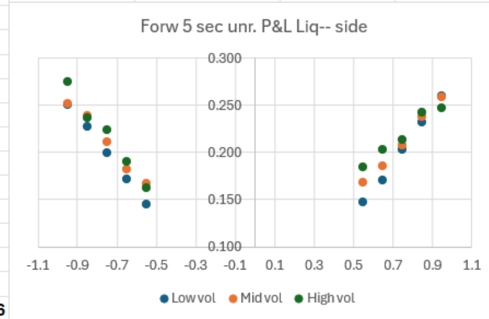
		Volatility	0.279	0.479	9.359	
Imbalance Buckets		**buckets**	0.029	0.279	0.479	
0.9	1	0.95	0.473	0.486	0.501	0.487
0.8	0.9	0.85	0.447	0.470	0.486	0.467
0.7	0.8	0.75	0.413	0.455	0.472	0.447
0.6	0.7	0.65	0.384	0.438	0.469	0.431
0.5	0.6	0.55	0.358	0.422	0.461	0.414
-0.5	-0.6	-0.55	0.358	0.418	0.463	0.413
-0.6	-0.7	-0.65	0.396	0.443	0.471	0.437
-0.7	-0.8	-0.75	0.417	0.455	0.482	0.452
-0.8	-0.9	-0.85	0.446	0.471	0.494	0.469
-0.9	-1	-0.95	0.469	0.485	0.495	0.483
			0.411	0.451	0.477	**0.447**



Year 2023 Results

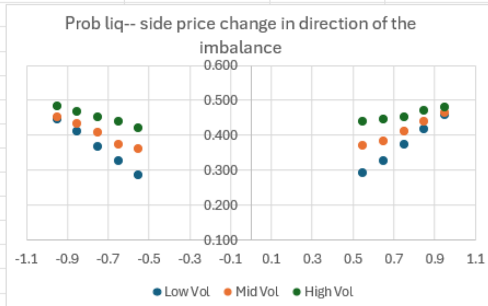
Forw 5 sec unr. P&L Liq-- side

		Volatility	0.149	0.262	8.151	
Imbalance Buckets		**Buckets**	0.017	0.149	0.262	
0.9	1	0.95	0.260	0.259	0.247	0.255
0.8	0.9	0.85	0.232	0.238	0.242	0.237
0.7	0.8	0.75	0.203	0.208	0.213	0.208
0.6	0.7	0.65	0.170	0.185	0.203	0.186
0.5	0.6	0.55	0.147	0.169	0.184	0.167
-0.5	-0.6	-0.55	0.145	0.167	0.162	0.158
-0.6	-0.7	-0.65	0.171	0.182	0.190	0.181
-0.7	-0.8	-0.75	0.200	0.211	0.223	0.211
-0.8	-0.9	-0.85	0.227	0.239	0.236	0.234
-0.9	-1	-0.95	0.251	0.251	0.275	0.259
			0.197	0.207	0.213	**0.206**



Prob of Liq-- price change in direction matching imbalance

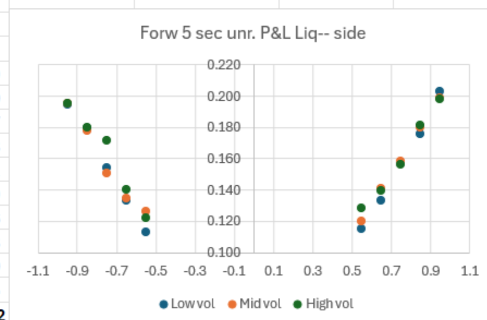
		Volatility	0.149	0.262	8.151	
Imbalance Buckets		**buckets**	0.017	0.149	0.262	
0.9	1	0.95	0.457	0.463	0.480	0.466
0.8	0.9	0.85	0.417	0.438	0.471	0.442
0.7	0.8	0.75	0.375	0.411	0.453	0.412
0.6	0.7	0.65	0.327	0.384	0.445	0.386
0.5	0.6	0.55	0.292	0.369	0.438	0.369
-0.5	-0.6	-0.55	0.287	0.360	0.421	0.358
-0.6	-0.7	-0.65	0.327	0.374	0.439	0.381
-0.7	-0.8	-0.75	0.368	0.409	0.452	0.409
-0.8	-0.9	-0.85	0.410	0.434	0.467	0.437
-0.9	-1	-0.95	0.444	0.453	0.483	0.460
			0.365	0.405	0.452	**0.407**



Year 2024 Results

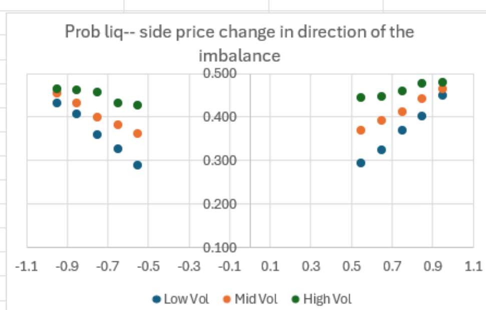
Forw 5 sec unr. P&L Liq-- side

		Volatility	0.129	0.230	8.196	
Imbalance Buckets		**Buckets**	0.019	0.129	0.230	
0.9	1	0.95	0.203	0.198	0.198	0.200
0.8	0.9	0.85	0.175	0.180	0.181	0.179
0.7	0.8	0.75	0.157	0.159	0.156	0.157
0.6	0.7	0.65	0.133	0.141	0.139	0.138
0.5	0.6	0.55	0.115	0.120	0.128	0.121
-0.5	-0.6	-0.55	0.113	0.126	0.122	0.120
-0.6	-0.7	-0.65	0.133	0.135	0.140	0.136
-0.7	-0.8	-0.75	0.154	0.150	0.172	0.158
-0.8	-0.9	-0.85	0.179	0.178	0.180	0.179
-0.9	-1	-0.95	0.195	0.195	0.196	0.195
			0.149	0.151	0.154	**0.152**



Prob of Liq-- price change in direction matching imbalance

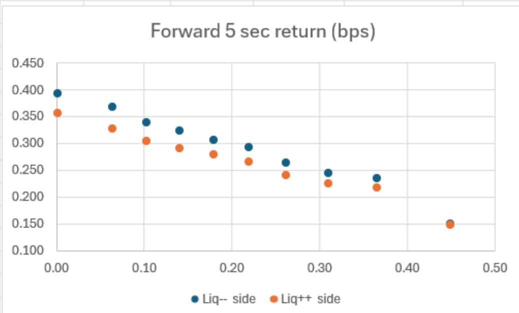
		Volatility	0.129	0.230	8.196	
Imbalance Buckets		**buckets**	0.019	0.129	0.230	
0.9	1	0.95	0.450	0.465	0.480	0.464
0.8	0.9	0.85	0.403	0.442	0.477	0.440
0.7	0.8	0.75	0.369	0.412	0.460	0.413
0.6	0.7	0.65	0.324	0.391	0.446	0.387
0.5	0.6	0.55	0.294	0.369	0.444	0.371
-0.5	-0.6	-0.55	0.288	0.360	0.427	0.360
-0.6	-0.7	-0.65	0.325	0.381	0.432	0.380
-0.7	-0.8	-0.75	0.358	0.400	0.457	0.404
-0.8	-0.9	-0.85	0.406	0.431	0.462	0.433
-0.9	-1	-0.95	0.432	0.455	0.464	0.450
			0.353	0.402	0.451	**0.402**

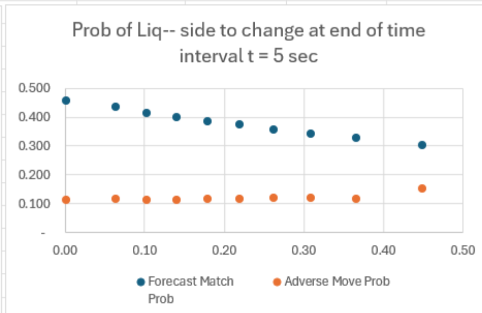


Appendix 4. Liq—normalized size factor analysis 2018-2024

Year 2018 Results

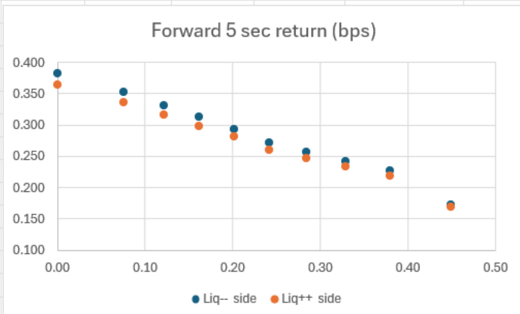
Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.45	6.30	0.90	48,830	0.149	0.148	14,756	7,341	0.302	0.150
0.37	0.45	0.80	48,829	0.233	0.217	15,923	5,576	0.326	0.114
0.31	0.37	0.70	48,830	0.245	0.225	16,647	5,792	0.341	0.119
0.26	0.31	0.60	48,829	0.264	0.241	17,328	5,867	0.355	0.120
0.22	0.26	0.50	48,830	0.292	0.266	18,320	5,714	0.375	0.117
0.18	0.22	0.40	48,829	0.306	0.278	18,819	5,603	0.385	0.115
0.14	0.18	0.30	48,829	0.322	0.290	19,526	5,503	0.400	0.113
0.10	0.14	0.20	48,830	0.339	0.304	20,225	5,502	0.414	0.113
0.06	0.10	0.10	48,829	0.368	0.326	21,227	5,570	0.435	0.114
0.00	0.06	0.00	48,830	0.393	0.356	22,290	5,489	0.456	0.112
			488,295	0.291	0.265	185,061	57,957	0.379	0.119

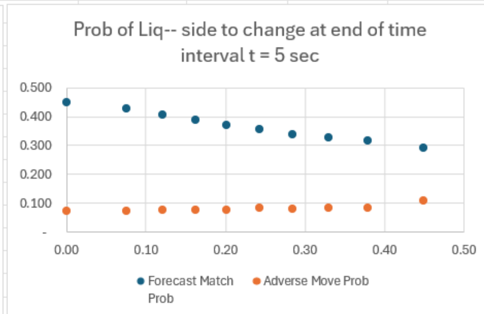




Year 2019 Results

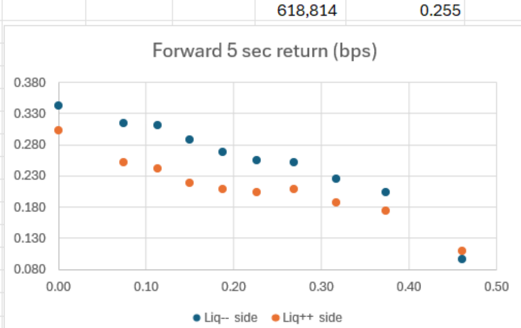
Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.45	3.62	0.90	43,587	0.173	0.168	12,693	4,736	0.291	0.109
0.38	0.45	0.80	43,587	0.227	0.219	13,730	3,594	0.315	0.082
0.33	0.38	0.70	43,587	0.242	0.232	14,315	3,559	0.328	0.082
0.28	0.33	0.60	43,586	0.257	0.247	14,784	3,498	0.339	0.080
0.24	0.28	0.50	43,587	0.271	0.260	15,574	3,585	0.357	0.082
0.20	0.24	0.40	43,587	0.292	0.280	16,170	3,351	0.371	0.077
0.16	0.20	0.30	43,586	0.312	0.298	16,965	3,362	0.389	0.077
0.12	0.16	0.20	43,587	0.331	0.315	17,731	3,314	0.407	0.076
0.08	0.12	0.10	43,587	0.352	0.336	18,641	3,232	0.428	0.074
0.00	0.08	0.00	43,587	0.382	0.363	19,585	3,116	0.449	0.071
			435,868	0.284	0.272	160,188	35,347	0.368	0.081

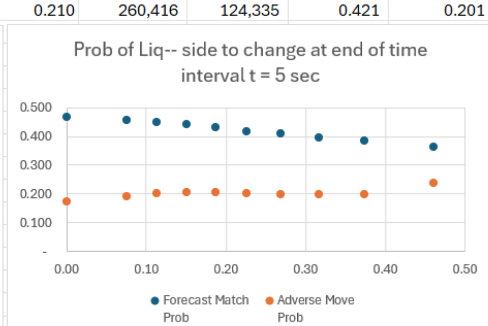




Year 2020 Results

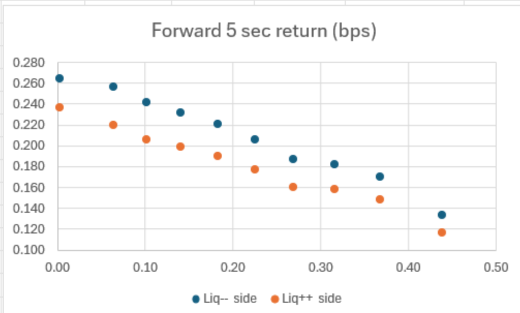
Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.46	4.79	0.90	61,882	0.095	0.109	22,374	14,663	0.362	0.237
0.37	0.46	0.80	61,881	0.203	0.173	23,787	12,298	0.384	0.199
0.32	0.37	0.70	61,881	0.225	0.187	24,522	12,265	0.396	0.198
0.27	0.32	0.60	61,882	0.251	0.208	25,250	12,316	0.408	0.199
0.23	0.27	0.50	61,881	0.255	0.203	25,822	12,577	0.417	0.203
0.19	0.23	0.40	61,881	0.268	0.209	26,589	12,585	0.430	0.203
0.15	0.19	0.30	61,882	0.288	0.218	27,311	12,640	0.441	0.204
0.11	0.15	0.20	61,881	0.310	0.241	27,702	12,529	0.448	0.202
0.08	0.11	0.10	61,881	0.314	0.252	28,143	11,813	0.455	0.191
0.00	0.08	0.00	61,882	0.341	0.303	28,916	10,649	0.467	0.172
			618,814	0.255	0.210	260,416	124,335	0.421	0.201

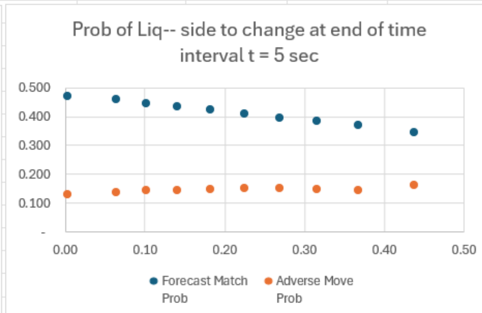




Year 2021 Results

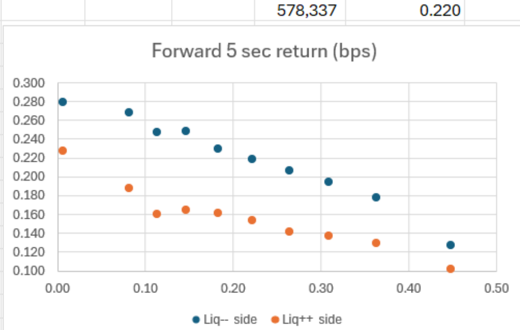
Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.44	3.53	0.90	57,059	0.133	0.116	19,779	9,244	0.347	0.162
0.37	0.44	0.80	57,058	0.170	0.148	21,130	8,285	0.370	0.145
0.32	0.37	0.70	57,058	0.182	0.158	21,904	8,409	0.384	0.147
0.27	0.32	0.60	57,058	0.187	0.160	22,596	8,709	0.396	0.153
0.23	0.27	0.50	57,059	0.206	0.177	23,311	8,710	0.409	0.153
0.18	0.23	0.40	57,058	0.220	0.190	24,083	8,515	0.422	0.149
0.14	0.18	0.30	57,058	0.231	0.199	24,789	8,243	0.434	0.144
0.10	0.14	0.20	57,058	0.241	0.206	25,435	8,145	0.446	0.143
0.06	0.10	0.10	57,058	0.256	0.220	26,213	7,778	0.459	0.136
0.00	0.06	0.00	57,059	0.264	0.237	26,915	7,437	0.472	0.130
			570,583	0.209	0.181	236,155	83,475	0.414	0.146

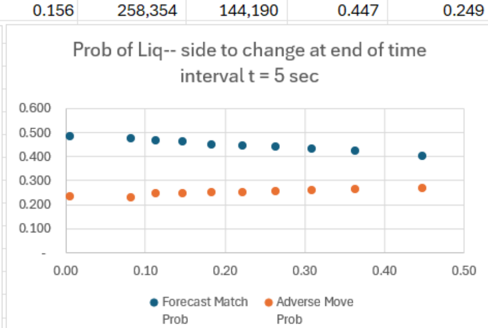




Year 2022 Results

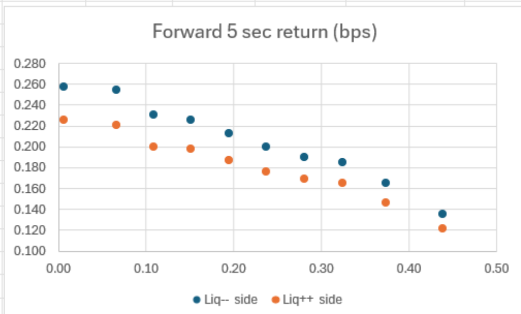
Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		Liq-- side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	Liq-- side	Liq++ side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.45	5.53	0.90	57,834	0.127	0.102	23,076	15,389	0.399	0.266
0.36	0.45	0.80	57,834	0.177	0.129	24,380	15,207	0.422	0.263
0.31	0.36	0.70	57,833	0.194	0.137	24,876	14,952	0.430	0.259
0.26	0.31	0.60	57,834	0.207	0.141	25,371	14,696	0.439	0.254
0.22	0.26	0.50	57,834	0.218	0.153	25,699	14,448	0.444	0.250
0.18	0.22	0.40	57,833	0.229	0.161	26,007	14,428	0.450	0.249
0.15	0.18	0.30	57,834	0.248	0.164	26,771	14,297	0.463	0.247
0.11	0.15	0.20	57,833	0.248	0.160	26,903	14,163	0.465	0.245
0.08	0.11	0.10	57,834	0.268	0.188	27,313	13,215	0.472	0.228
0.01	0.08	0.00	57,834	0.279	0.228	27,958	13,395	0.483	0.232
			578,337	0.220	0.156	258,354	144,190	0.447	0.249

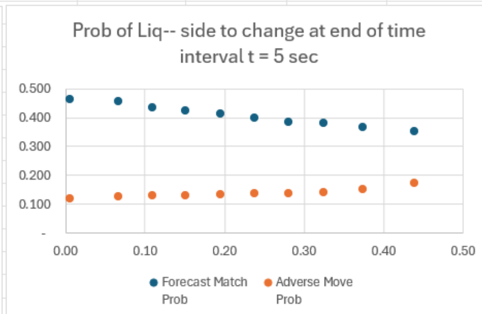




Year 2023 Results

Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.44	2.95	0.90	52,111	0.135	0.121	18,311	8,955	0.351	0.172
0.37	0.44	0.80	52,110	0.165	0.146	19,178	7,822	0.368	0.150
0.32	0.37	0.70	52,111	0.185	0.165	19,761	7,412	0.379	0.142
0.28	0.32	0.60	52,110	0.190	0.169	20,096	7,185	0.386	0.138
0.24	0.28	0.50	52,111	0.200	0.176	20,706	7,124	0.397	0.137
0.19	0.24	0.40	52,110	0.213	0.187	21,525	6,871	0.413	0.132
0.15	0.19	0.30	52,110	0.225	0.197	22,070	6,854	0.424	0.132
0.11	0.15	0.20	52,111	0.231	0.200	22,673	6,818	0.435	0.131
0.07	0.11	0.10	52,110	0.254	0.220	23,700	6,559	0.455	0.126
0.01	0.07	0.00	52,111	0.258	0.226	24,224	6,196	0.465	0.119
			521,105	0.206	0.181	212,244	71,796	0.407	0.138





Year 2024 Results

Norm Liq-- Buckets				Forward 5 sec unr. P&L (bps)		**Liq--** side: price change direction count/prob at the end of time interval t = 5 sec (from event time)			
From	To	Decile	Case Count	**Liq--** side	**Liq++** side	Forecast Match Count	Adverse Move Count	Forecast Match Prob	Adverse Move Prob
0.45	3.31	0.90	46,480	0.099	0.087	16,588	9,217	0.357	0.198
0.38	0.45	0.80	46,480	0.124	0.105	17,109	8,140	0.368	0.175
0.34	0.38	0.70	46,479	0.129	0.109	17,389	7,888	0.374	0.170
0.30	0.34	0.60	46,480	0.134	0.111	17,642	7,809	0.380	0.168
0.26	0.30	0.50	46,479	0.141	0.116	18,129	7,564	0.390	0.163
0.22	0.26	0.40	46,480	0.160	0.133	18,910	7,183	0.407	0.155
0.18	0.22	0.30	46,479	0.167	0.139	19,238	7,106	0.414	0.153
0.14	0.18	0.20	46,480	0.175	0.145	19,971	7,078	0.430	0.152
0.09	0.14	0.10	46,479	0.186	0.150	20,600	7,254	0.443	0.156
0.00	0.09	0.00	46,480	0.200	0.169	21,267	6,625	0.458	0.143
			464,796	0.152	0.126	186,843	75,864	0.402	0.163